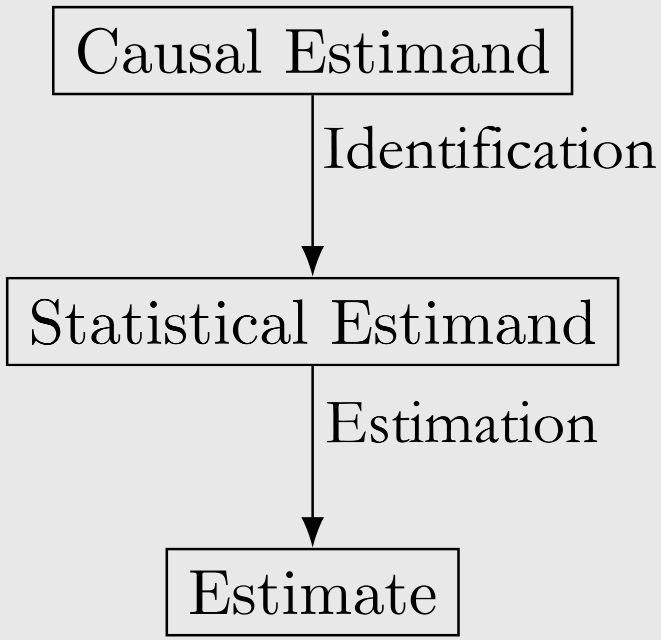
Causal Models

Brady Neal

causalcourse.com

The Identification-Estimation Flowchart



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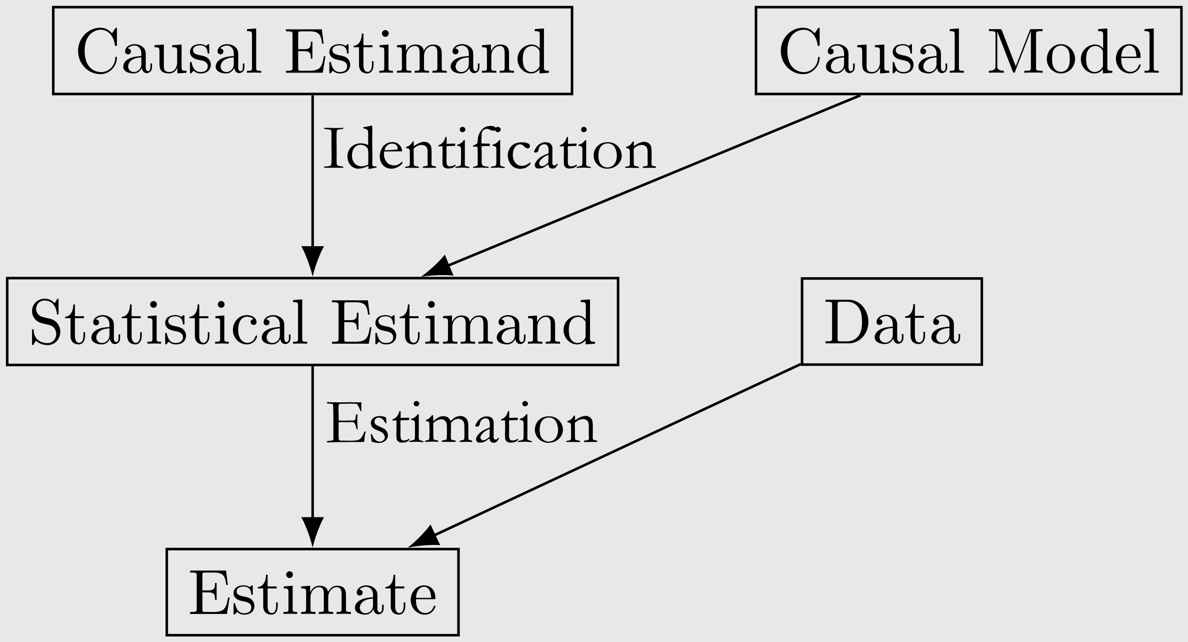

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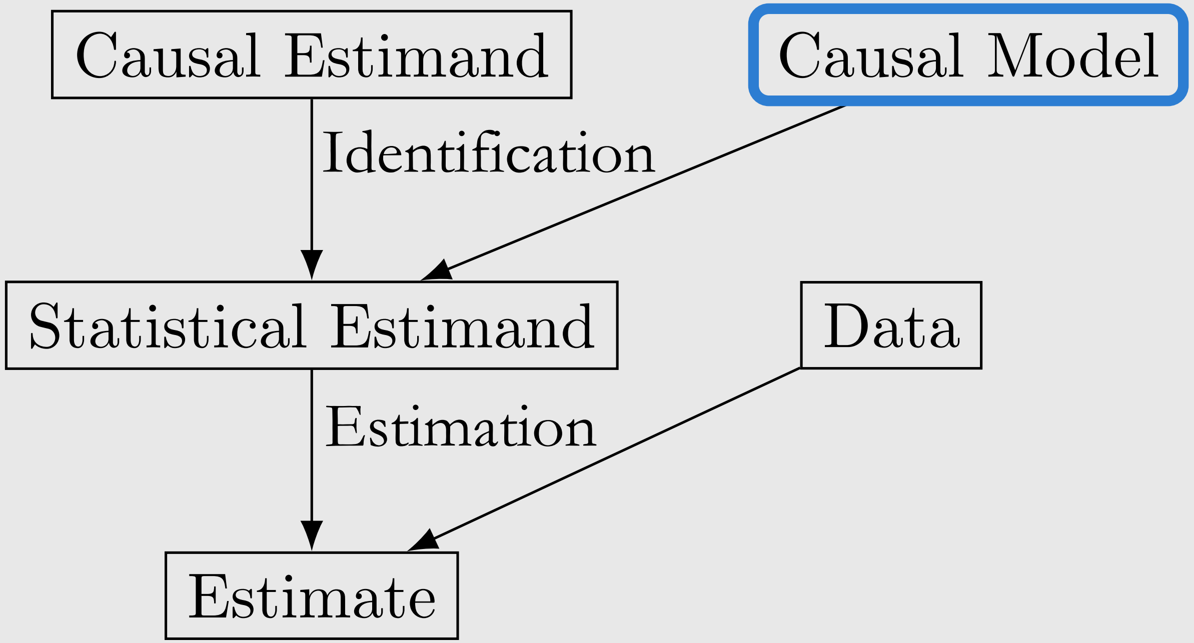

The *do*-operator

Main assumption: modularity

Backdoor adjustment

Structural causal models

A complete example with estimation

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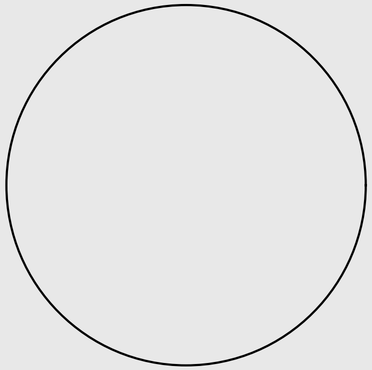
Conditioning vs. intervening

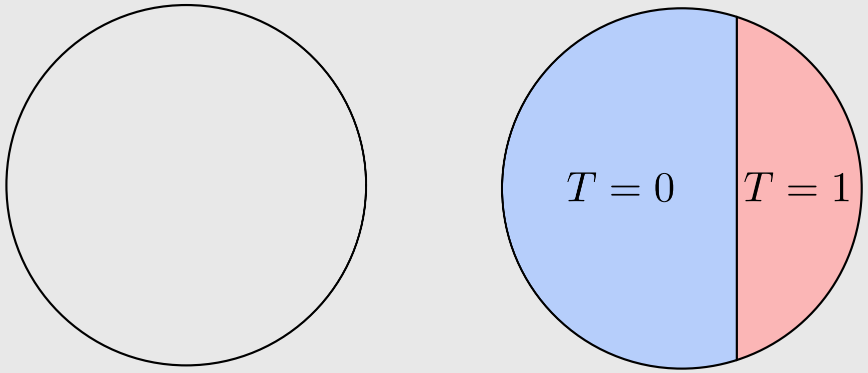
Conditioning vs. intervening

Population



Conditioning vs. intervening

Population　　Subpopulations



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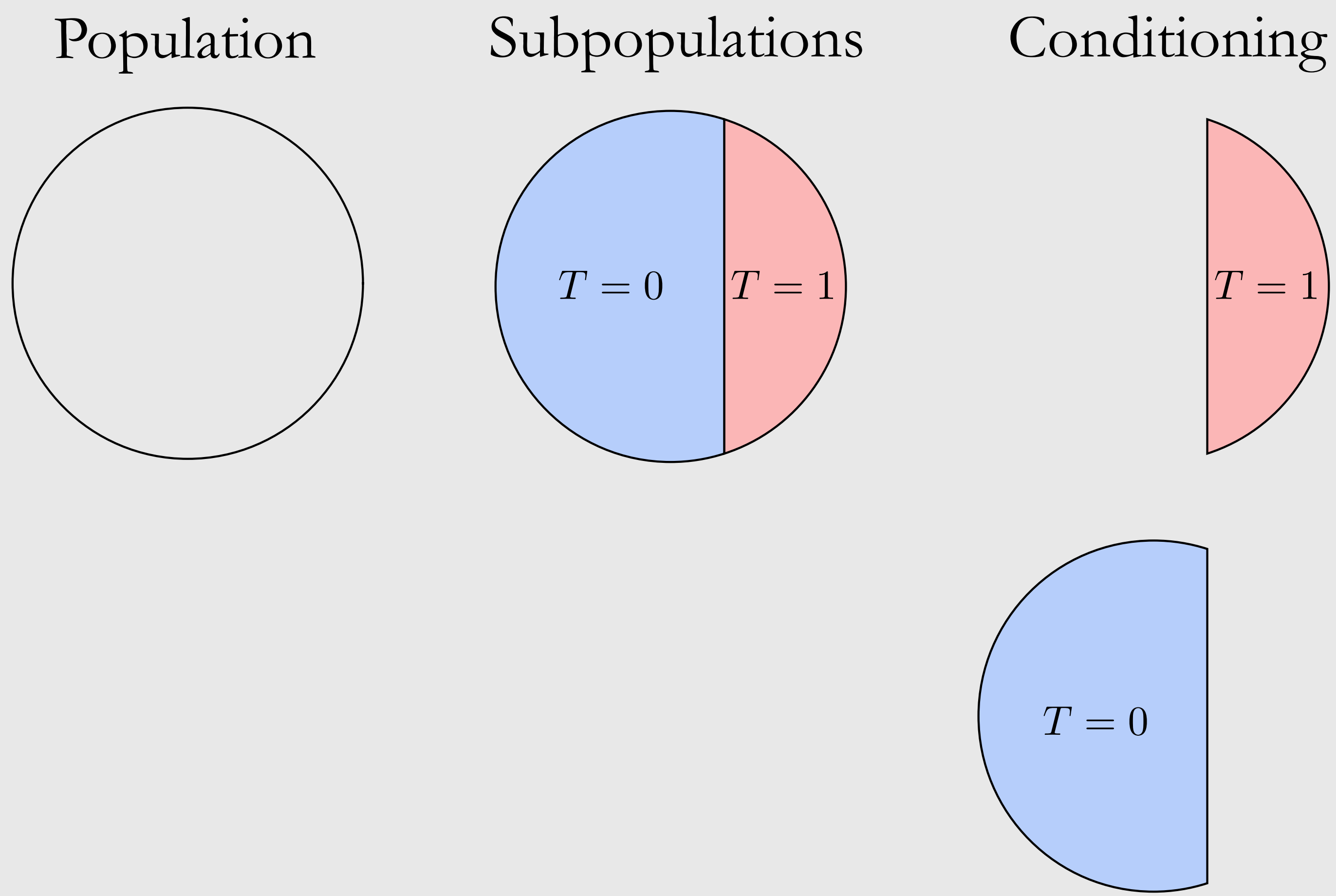

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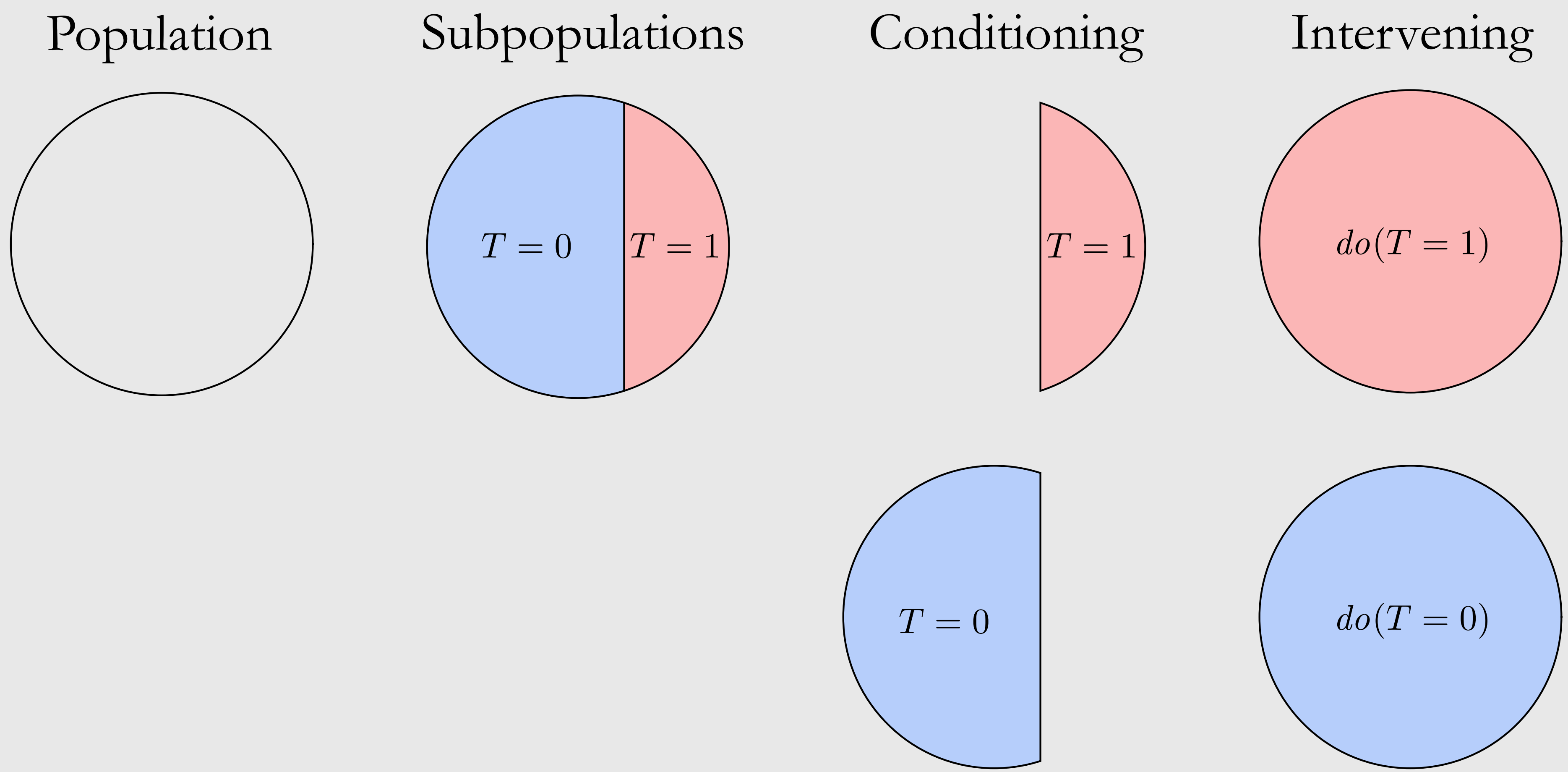

Some notation and terminology

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Interventional distributions:

$$P(Y(t) = y)$$

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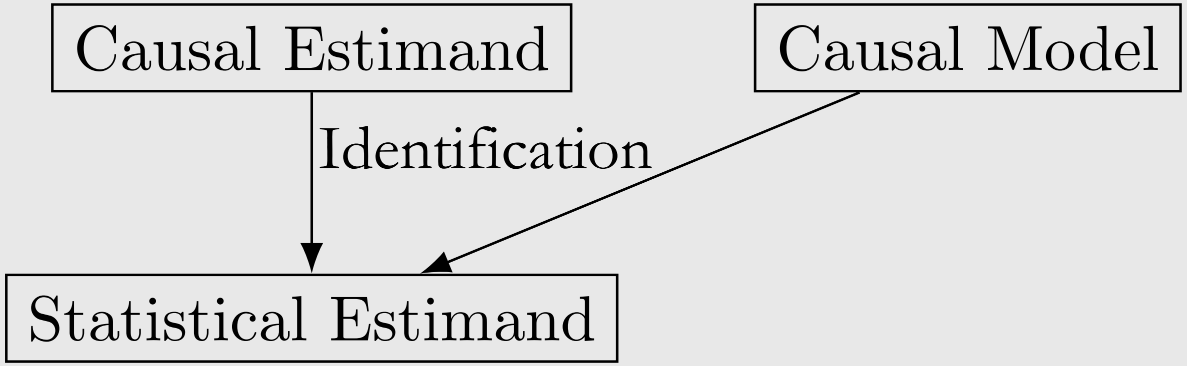
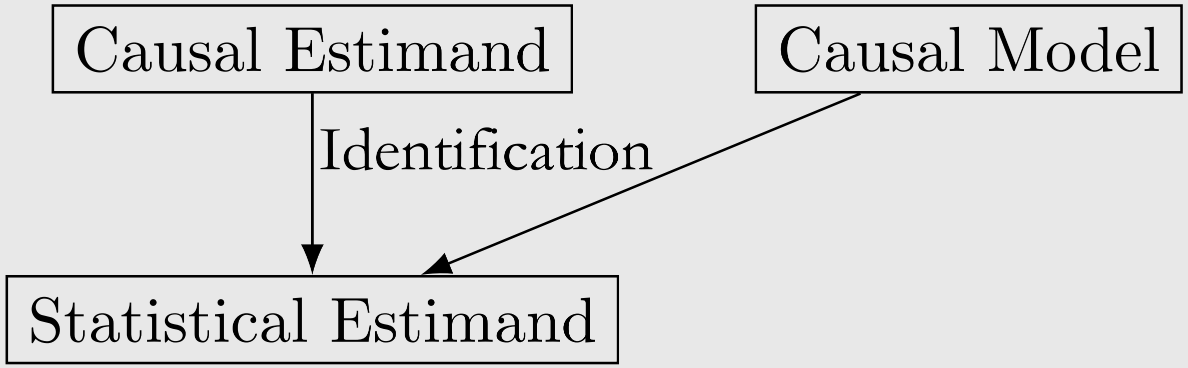
$P(Y \mid T = t)$	$P(Y \mid do(T = t), X = x)$

Identifiability



Identifiability

$P(y \mid do(t))$



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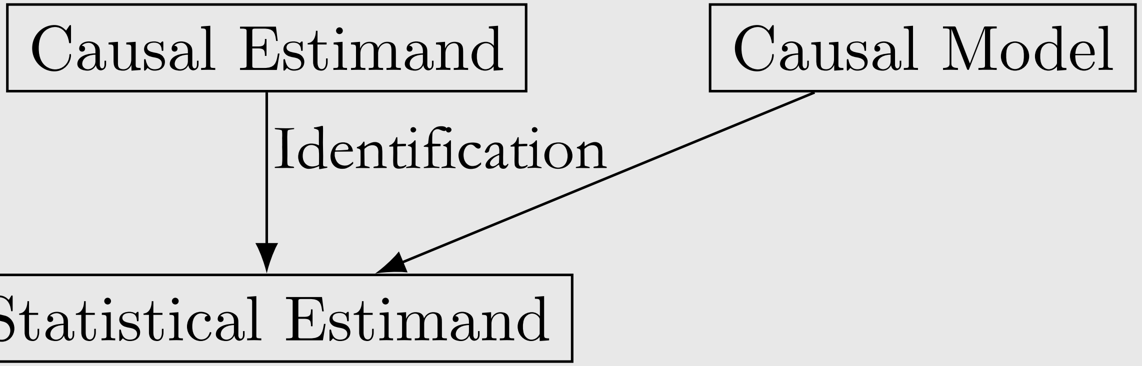

Identifiability

Accessible via experiment

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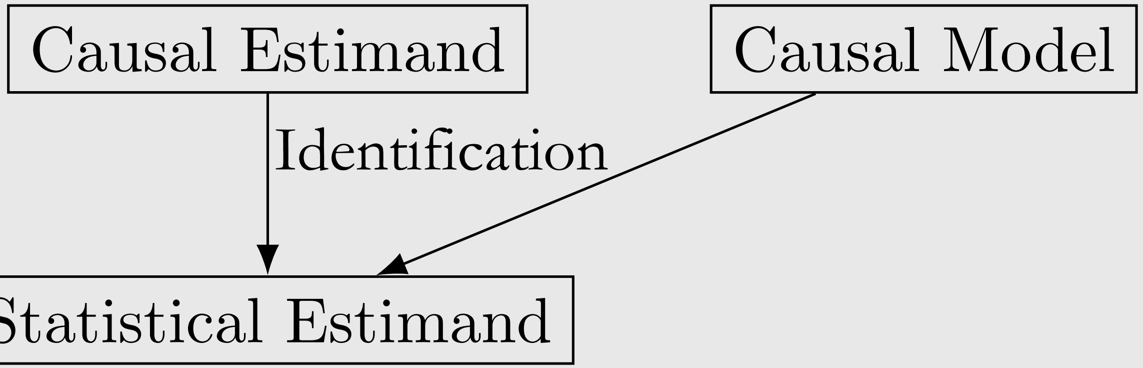

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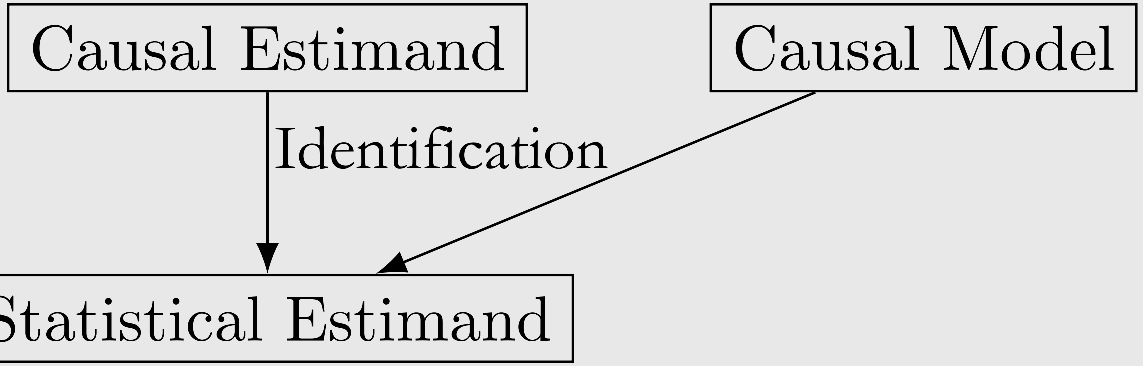

$P(y \mid t)$

Accessible via observational data

Identifiability

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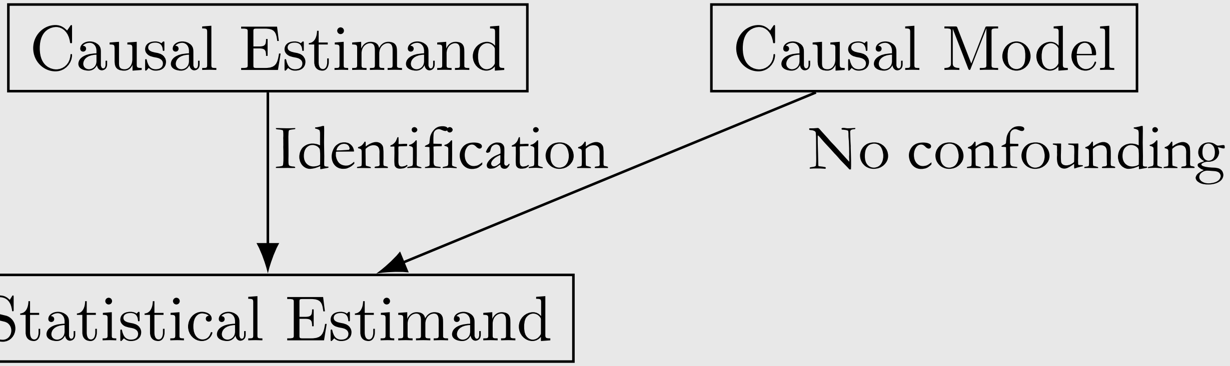

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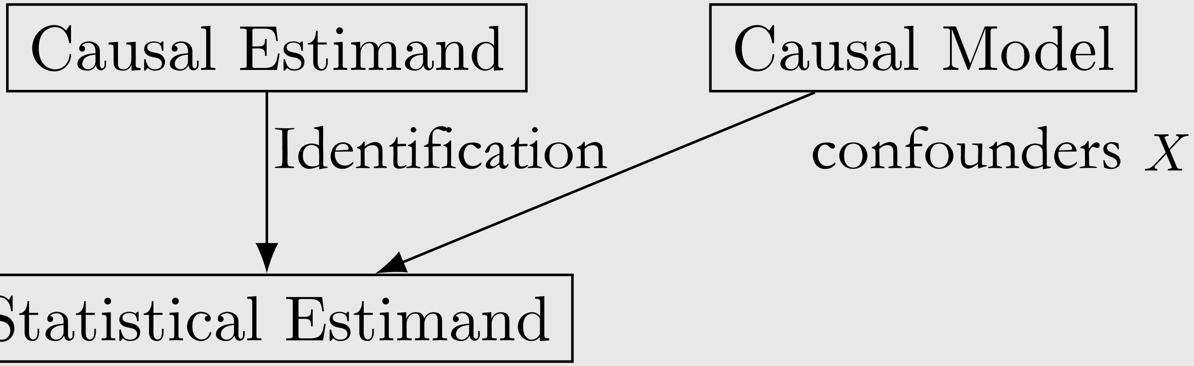

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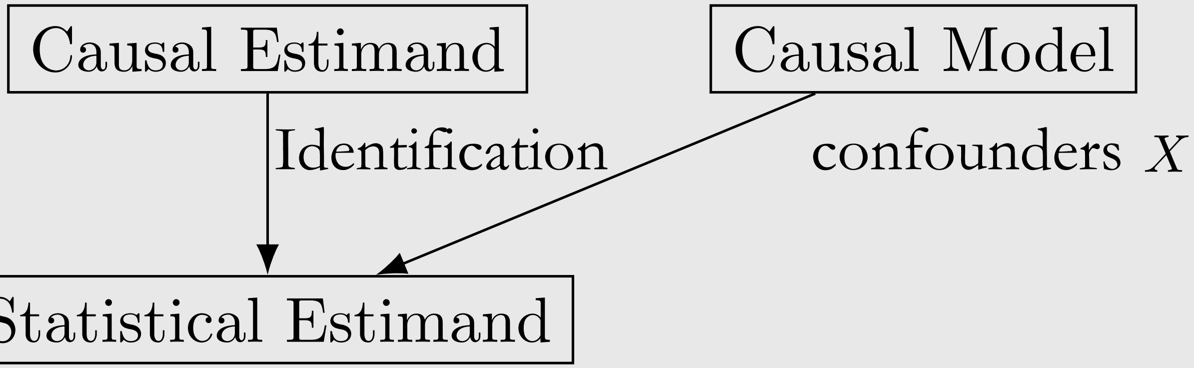

$\mathbb{E}_X[P(y \mid t, X)]$

Accessible via observational data

The *do*-operator

Main assumption: modularity

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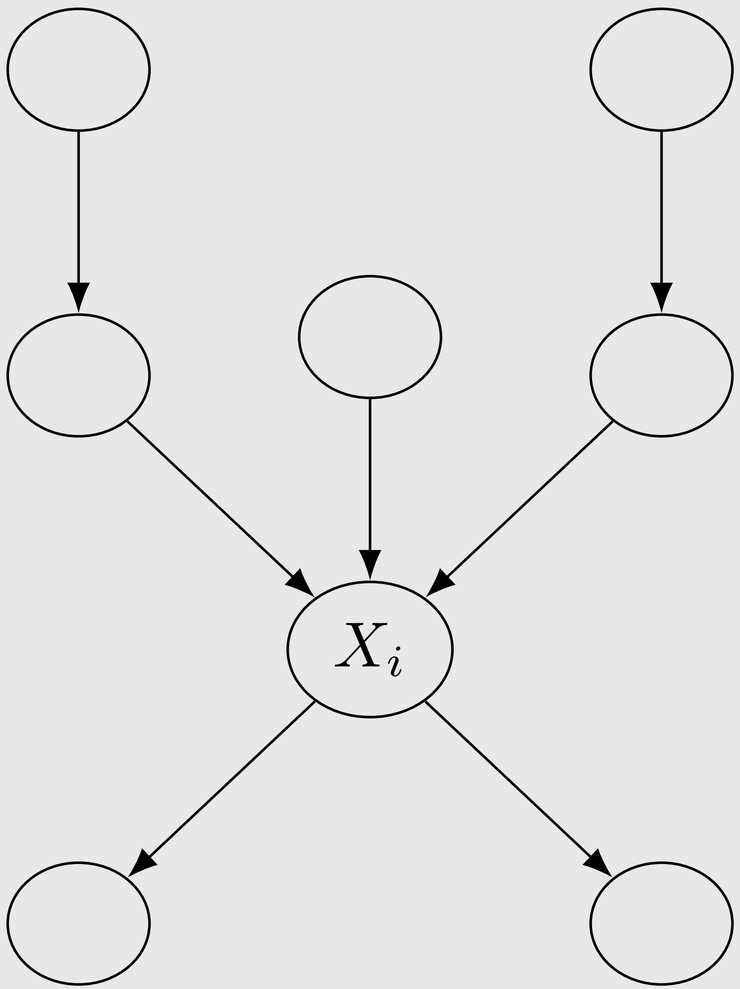
A complete example with estimation

Causal mechanism

$P(x_i \mid \text{pa}_i)$



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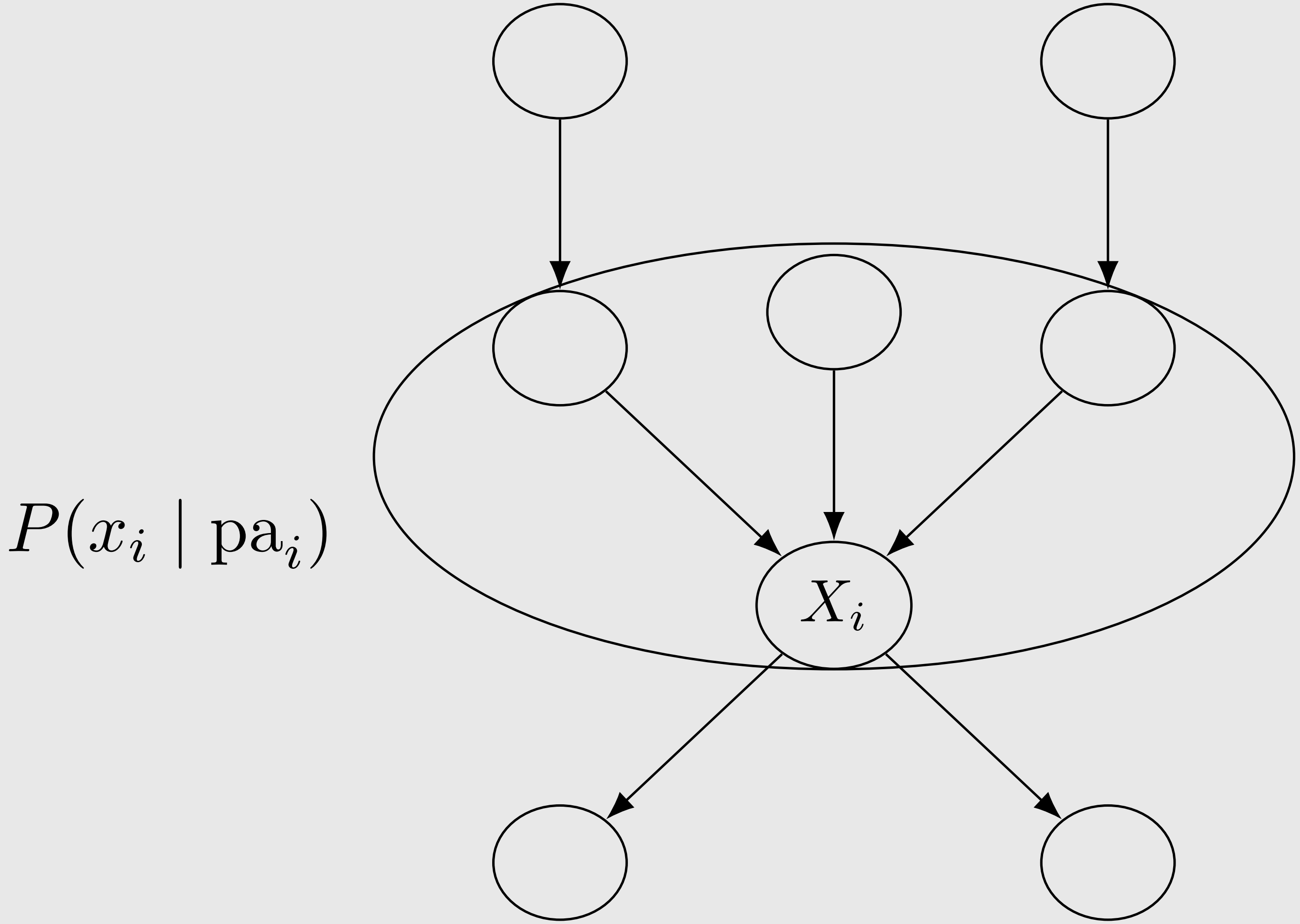

Modularity assumption

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If we intervene on a node X_i, then only the mechanism $P(x_i \mid \mathrm{pa}_i)$ changes. All other mechanisms $P(x_j \mid \mathrm{pa}_j)$ where $i \neq j$ remain unchanged.

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In other words, the causal mechanisms are **modular**.

Many names: independent mechanisms, autonomy, invariance, etc.

Modularity assumption: more formal

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If we intervene on a set of nodes $S \subseteq [n]$, setting them to constants, then for all i, we have the following:

1. If $i \notin S$, then $P(x_i \mid \mathrm{pa}_i)$ remains unchanged.
2. If $i \in S$, then $P(x_i \mid \mathrm{pa}_i) = 1$ if x_i is the value that X_i was set to by the intervention; otherwise, $P(x_i \mid \mathrm{pa}_i) = 0$.

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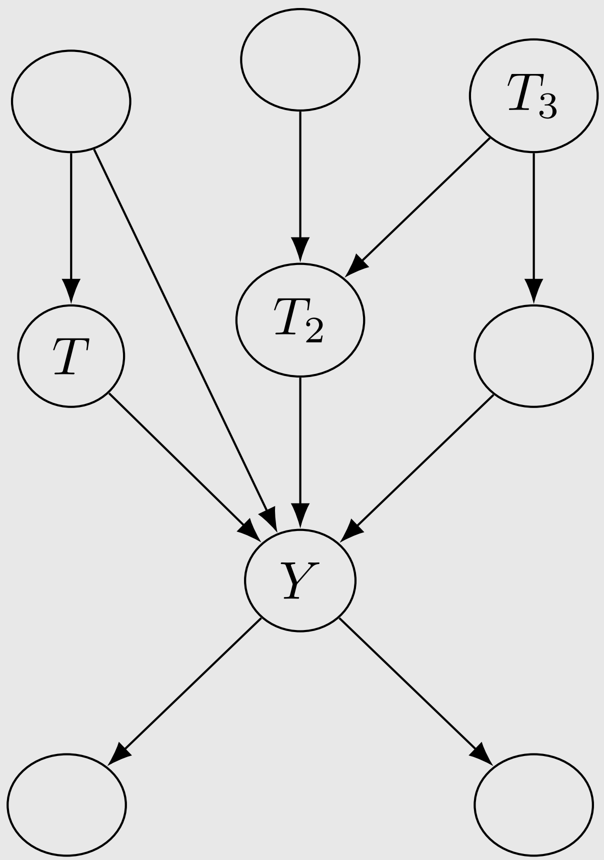
consistent with the intervention

Manipulated graphs

Observational data



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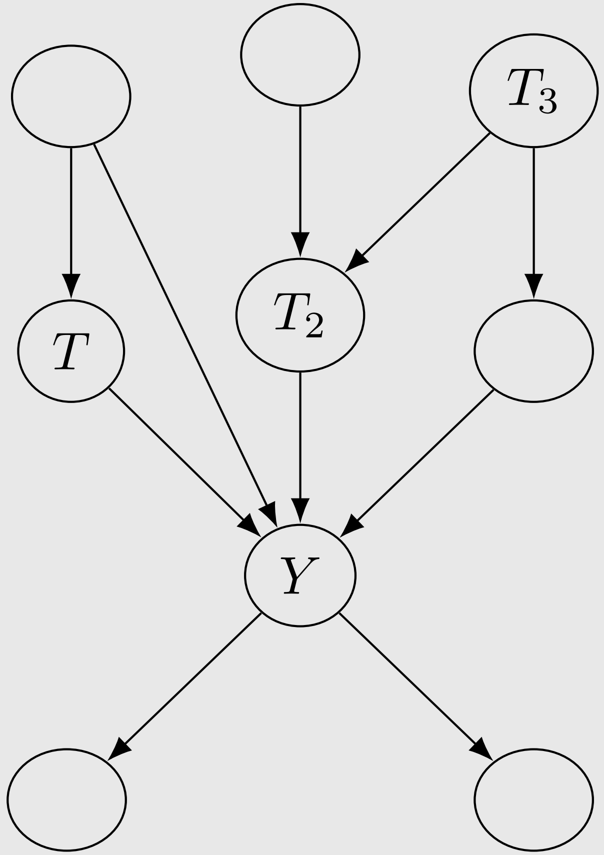
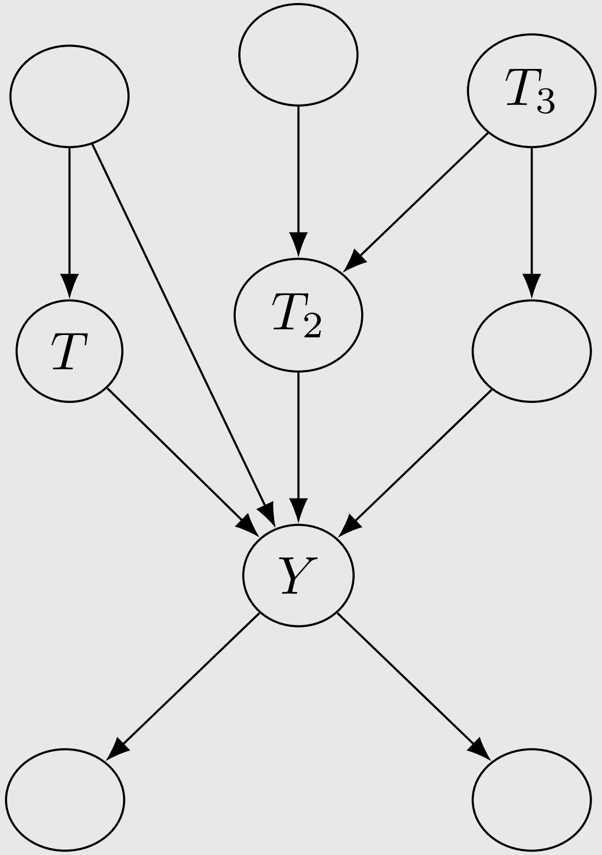


Interventional data



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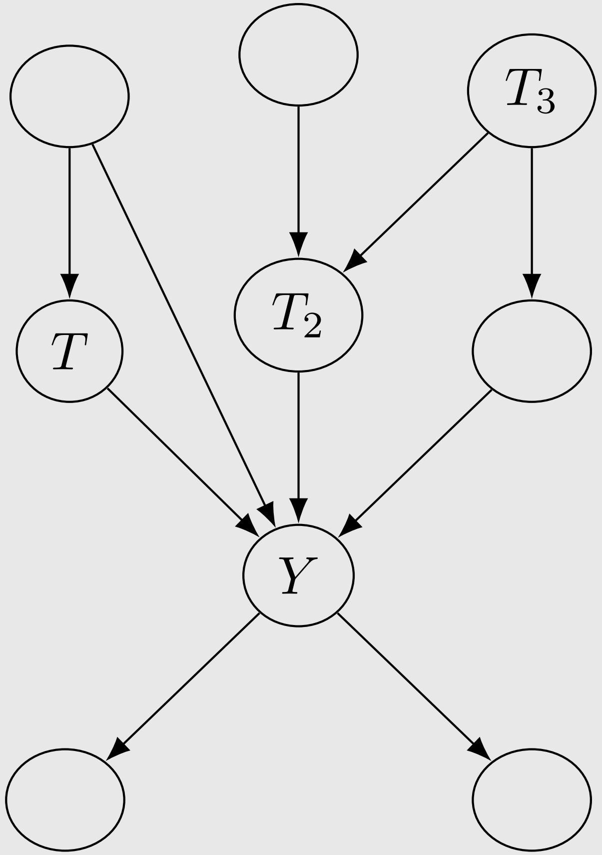

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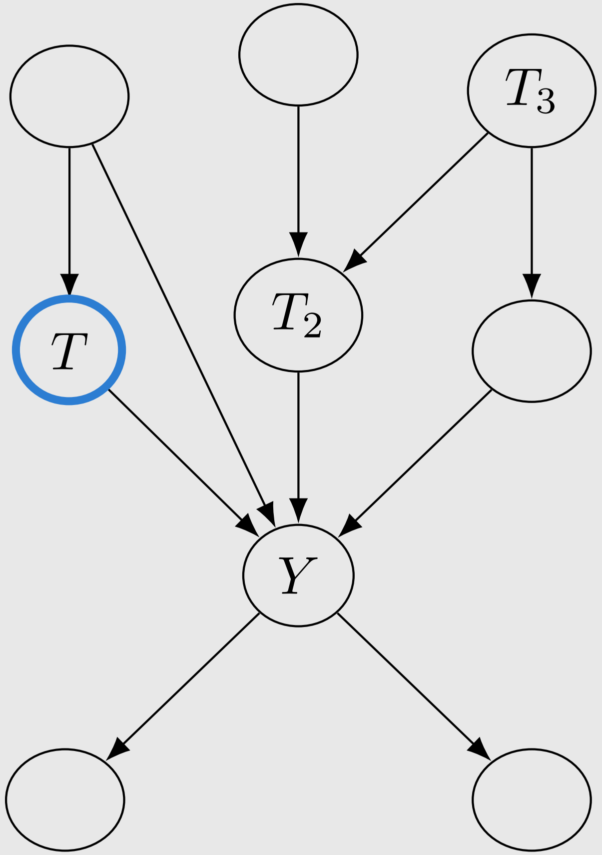

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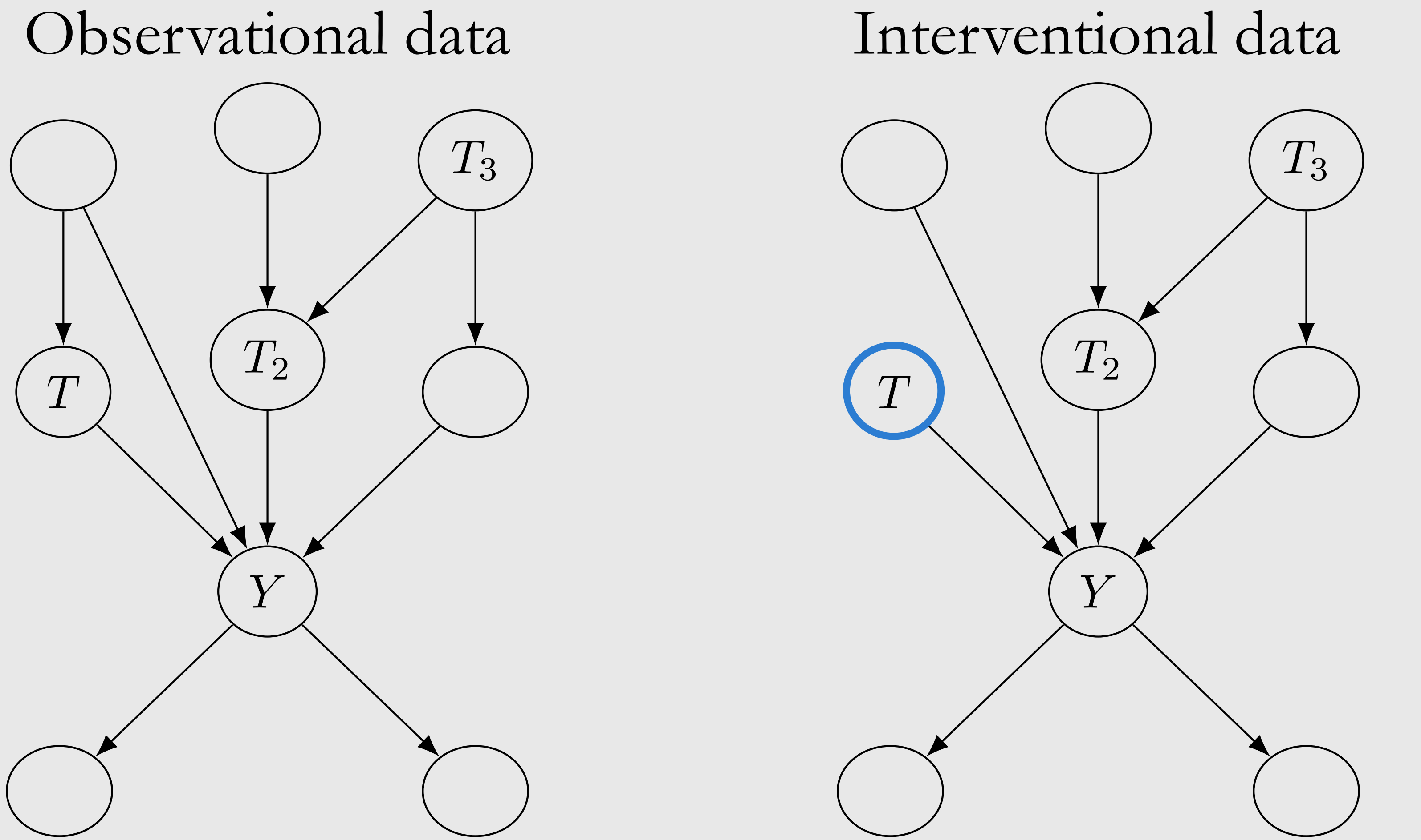

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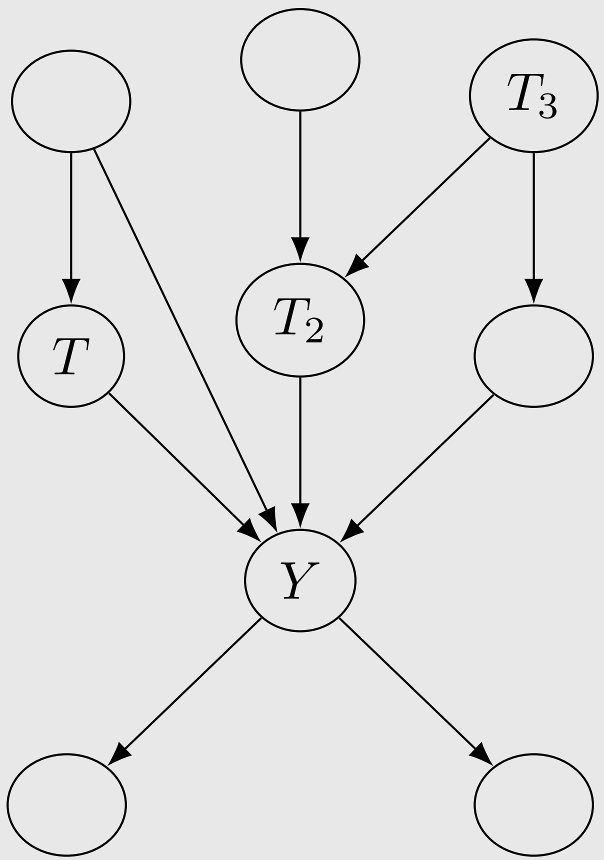

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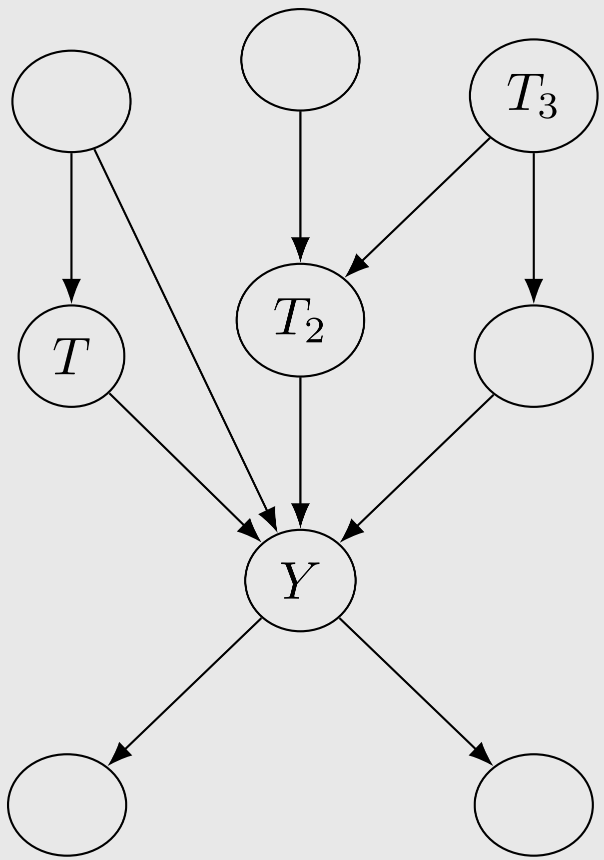

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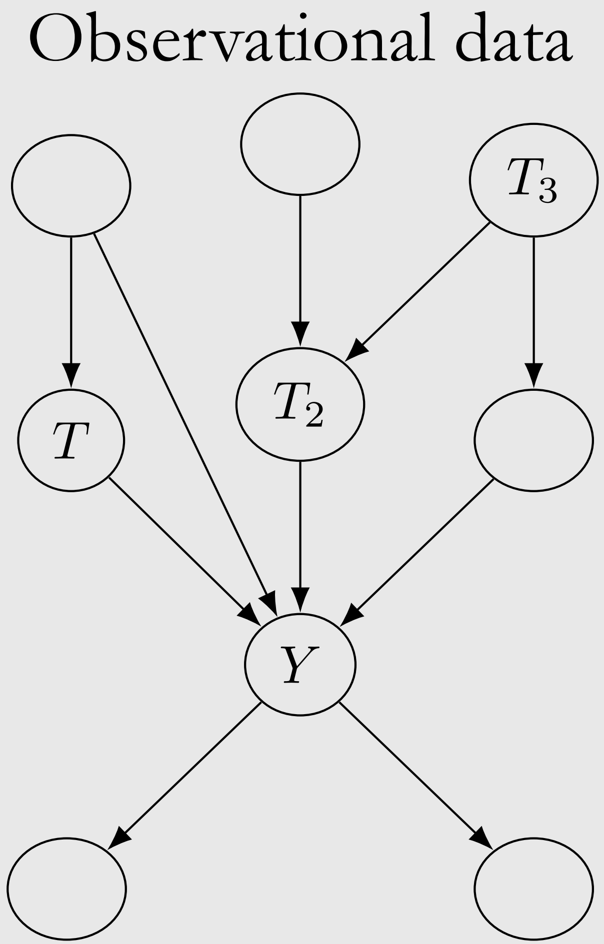

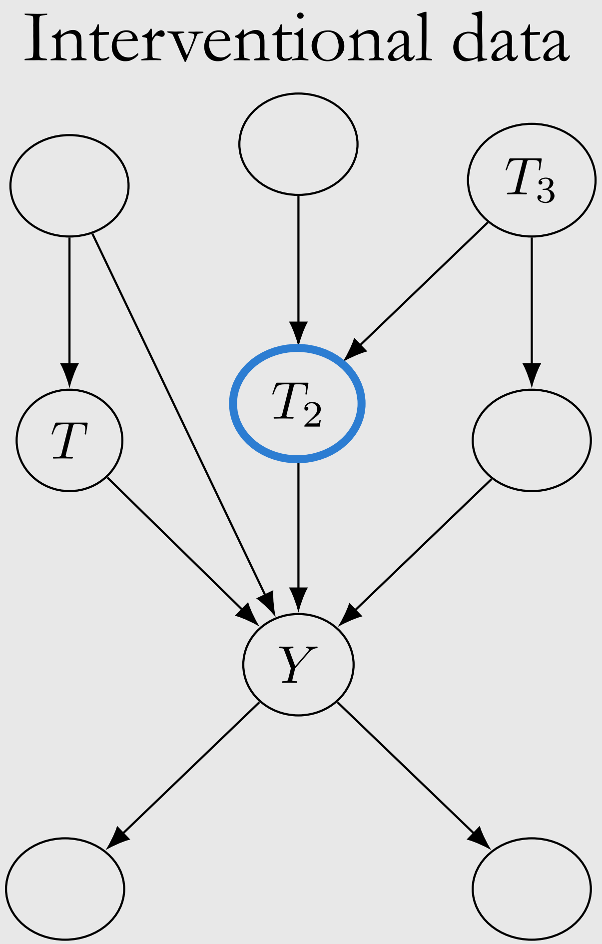

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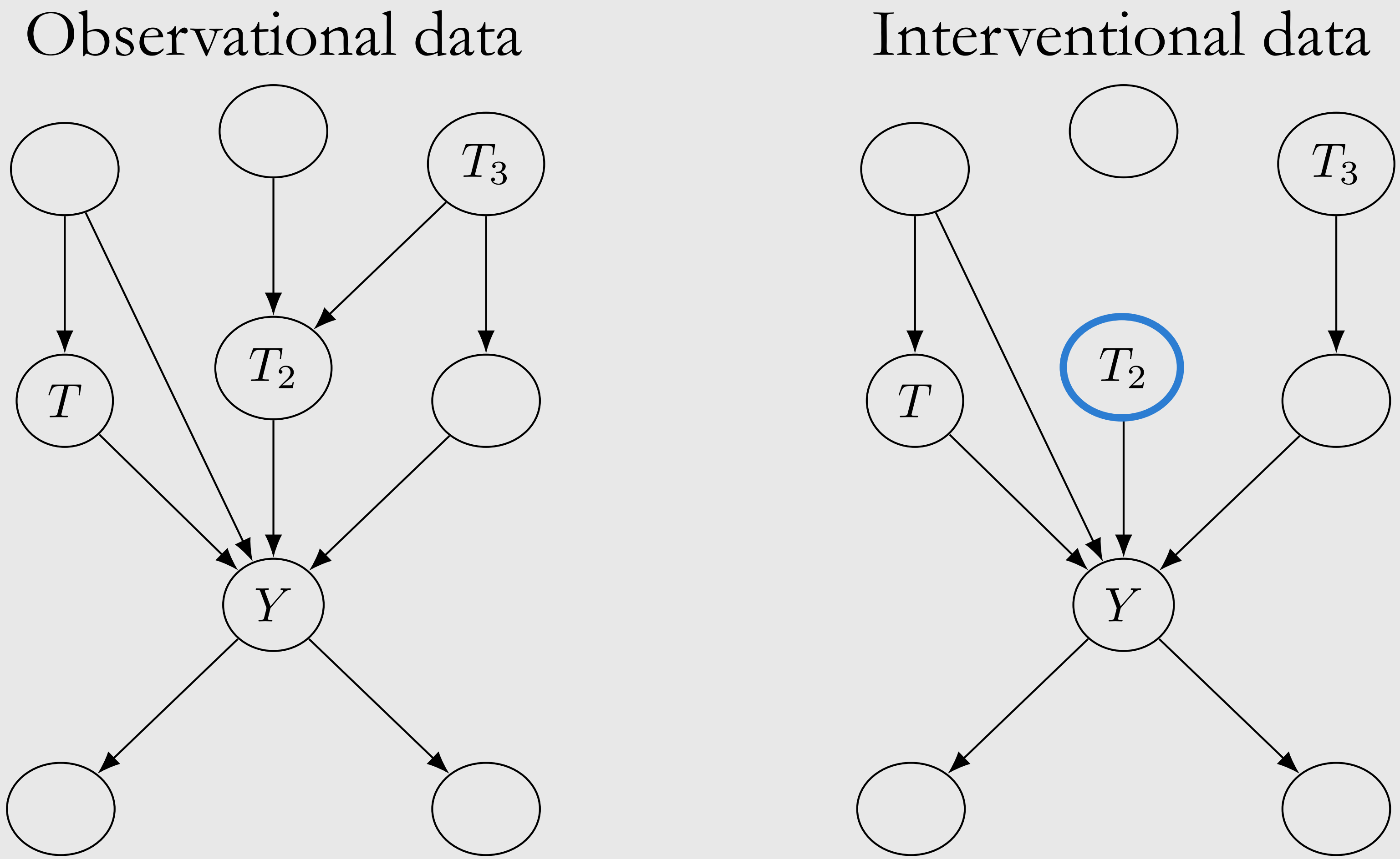
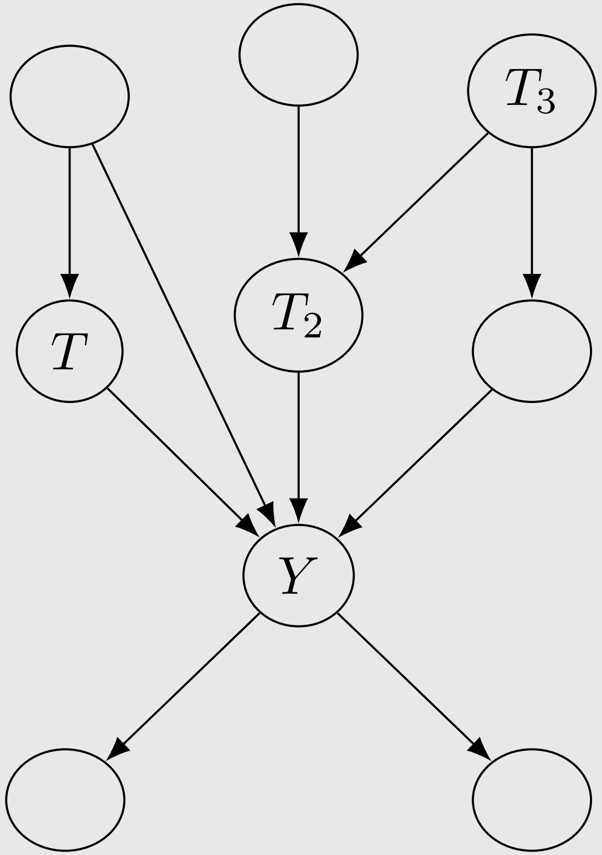


What would it mean if modularity is violated?



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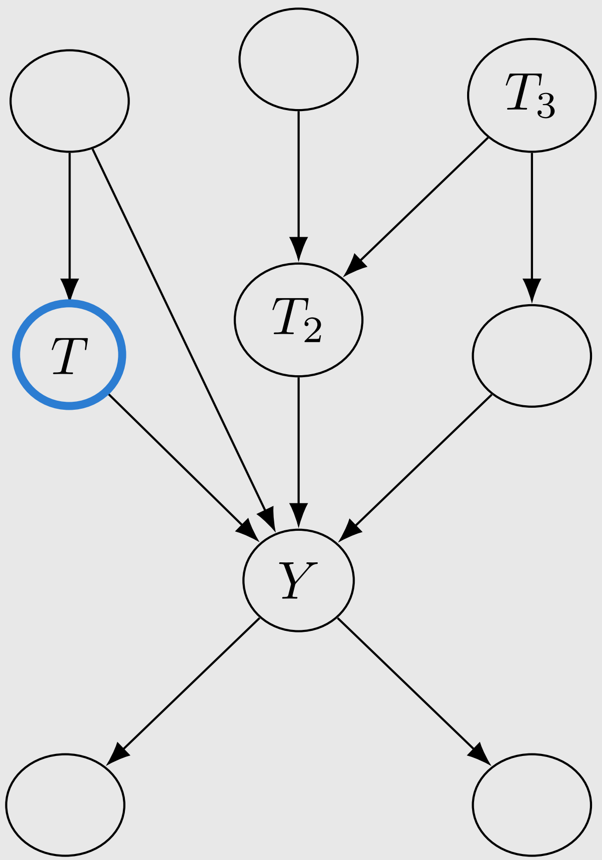
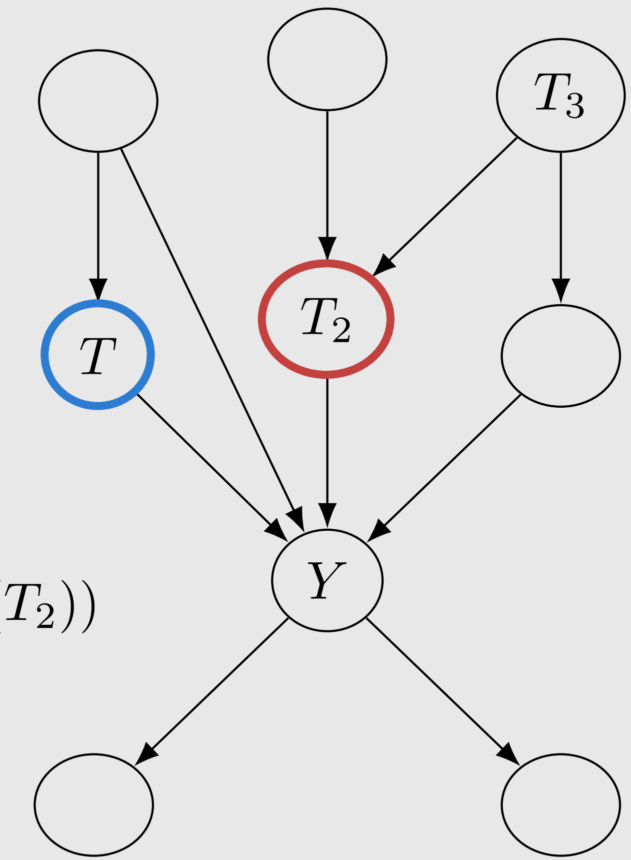
Intervention on T not only changes $P(T \mid \mathrm{pa}(T))$



What would it mean if modularity is violated?

Intervention on T not only changes $P(T \mid \text{pa}(T))$

but also changes other mechanisms such as $P(T_2 \mid \text{pa}(T_2))$



Truncated factorization

Recall the Bayesian network factorization:

$$P(x_1, \ldots, x_n) = \prod_i P(x_i \mid \mathrm{pa}_i)$$

Truncated factorization

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$$P(x_1, \ldots, x_n \mid do(S = s)) = \prod_i P(x_i \mid \text{pa}_i)$$

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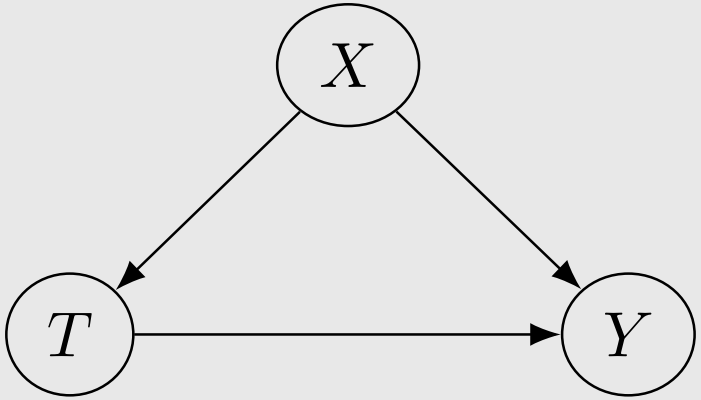
Otherwise,

$$P(x_1, \dots, x_n \mid do(S = s)) = 0$$

Simple identification via truncated factorization

Goal: identify $P(y \mid do(t))$



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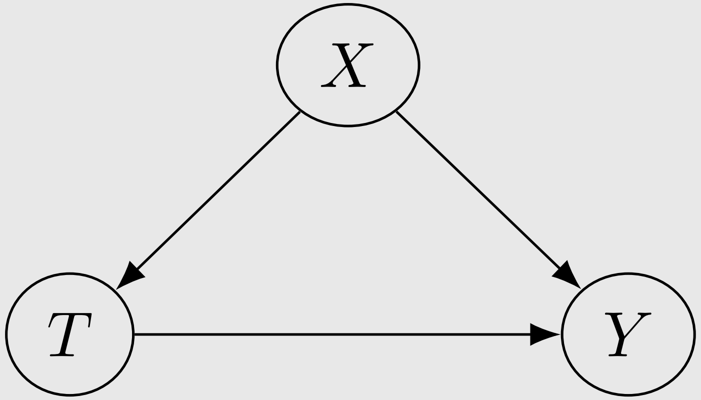

Bayesian network factorization: $P(y, t, x) = P(x)\, P(t \mid x)\, P(y \mid t, x)$

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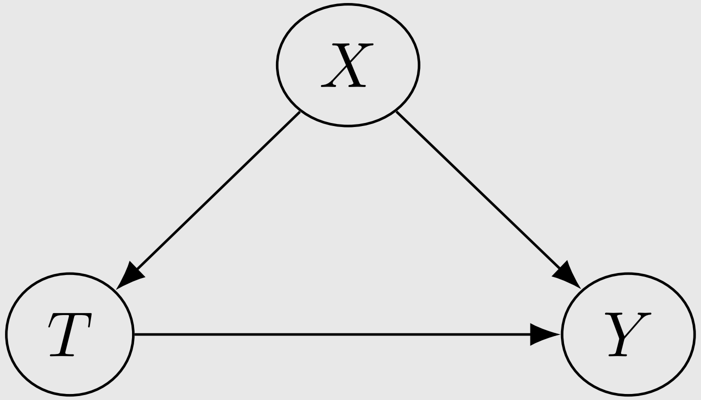

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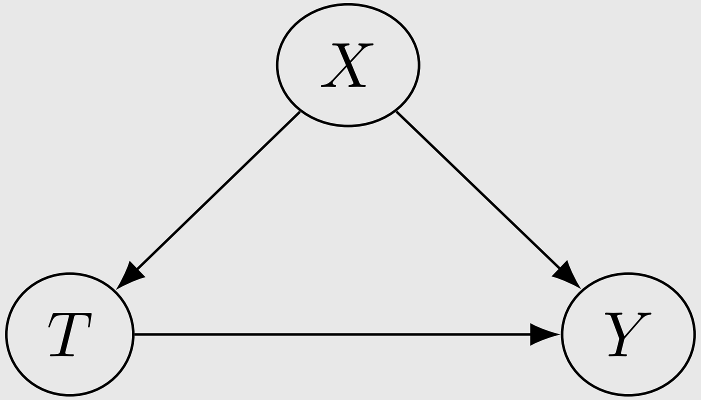

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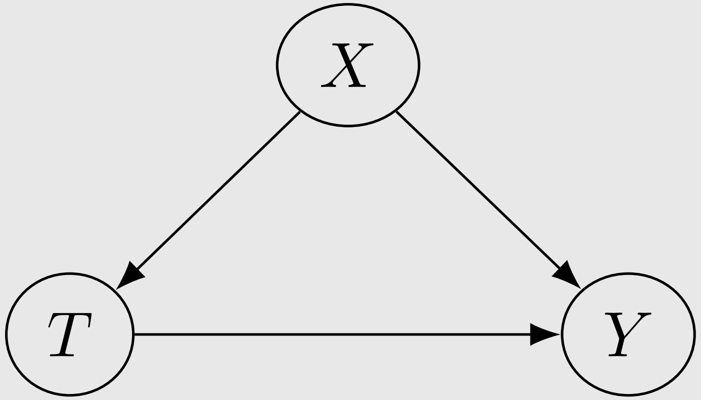
Marginalize: $P(y \mid do(t)) = \sum_x P(y \mid t, x)\, P(x)$

Association vs. causation revisited

$$P(y \mid do(t)) = \sum_x P(y \mid t, x)\, P(x)$$



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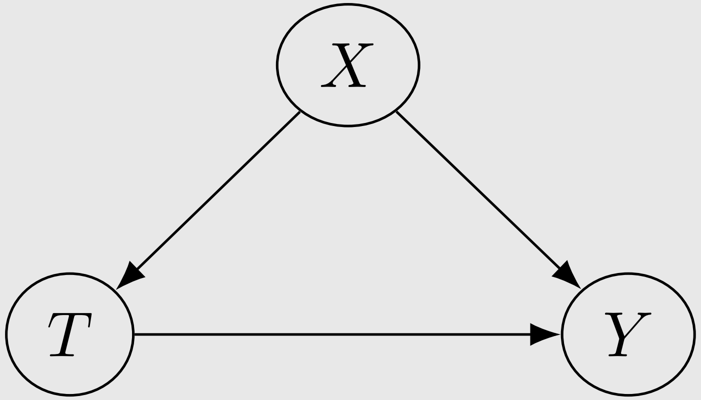

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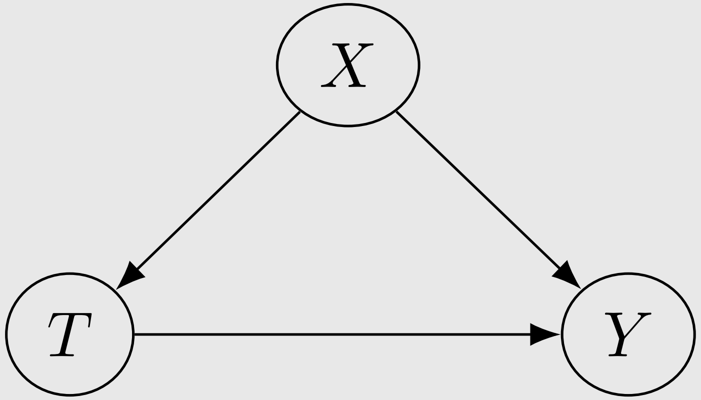

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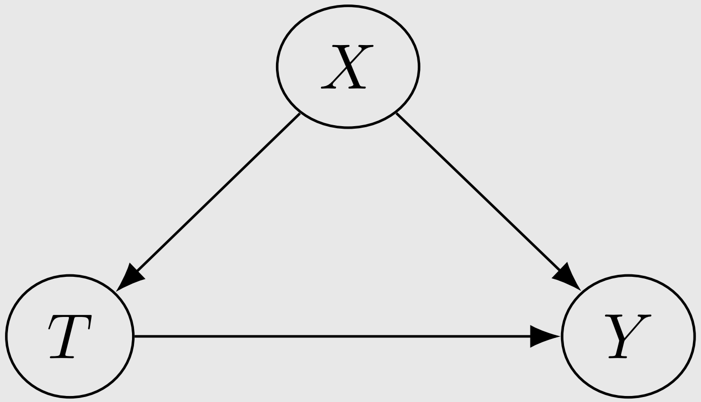

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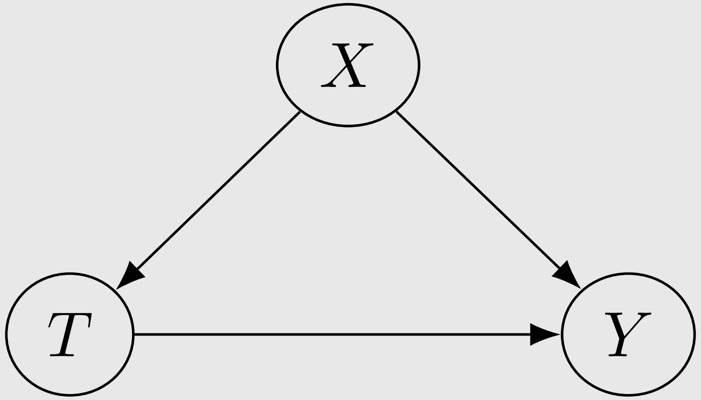

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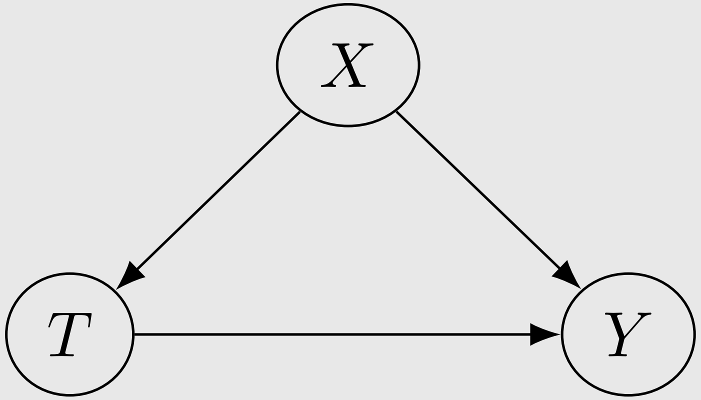

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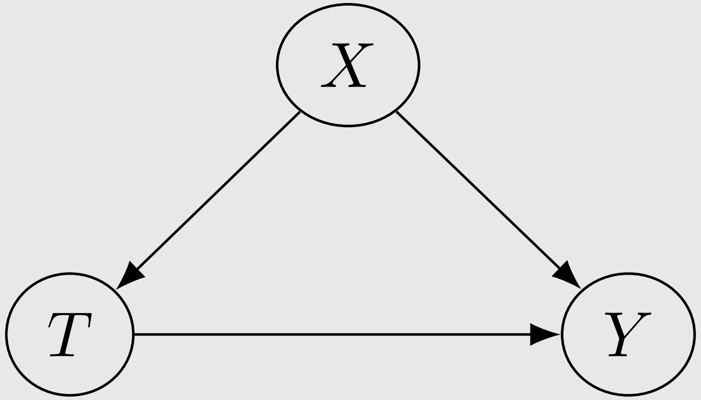

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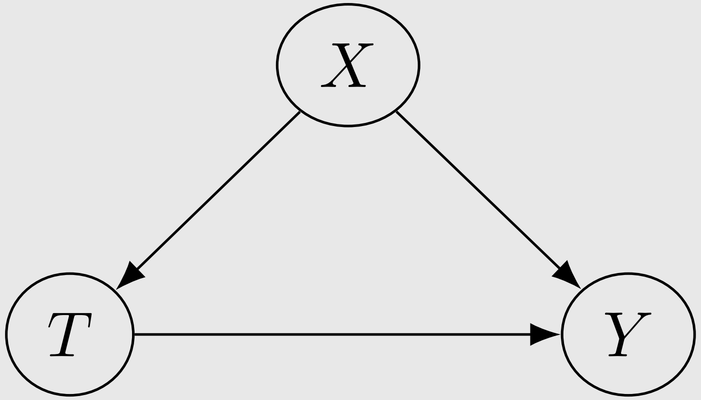

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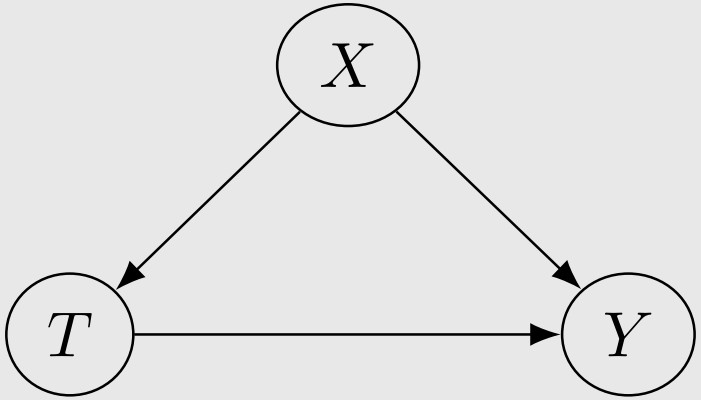

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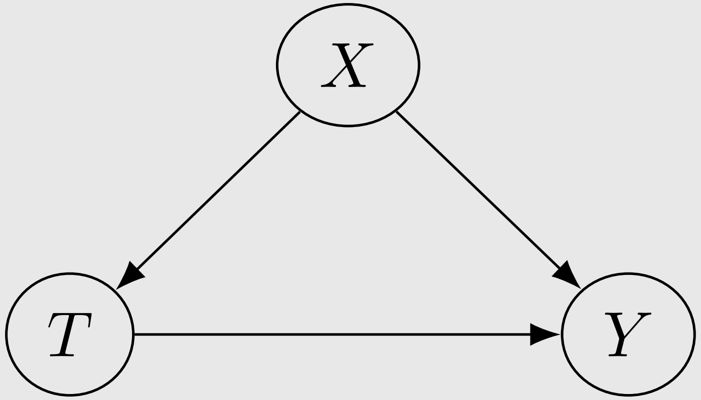
Association vs. causation revisited

$$\underline{P(y \mid do(t))} = \sum_x P(y \mid t, x)\, \underline{P(x)}$$



$$\underline{P(y \mid do(t))} \neq \underline{P(y \mid t)}$$

$$\sum_x P(y \mid t, x)\, \underline{P(x \mid t)} = \sum_x P(y, x \mid t)$$

$$= \underline{P(y \mid t)}$$

The *do*-operator

Main assumption: modularity

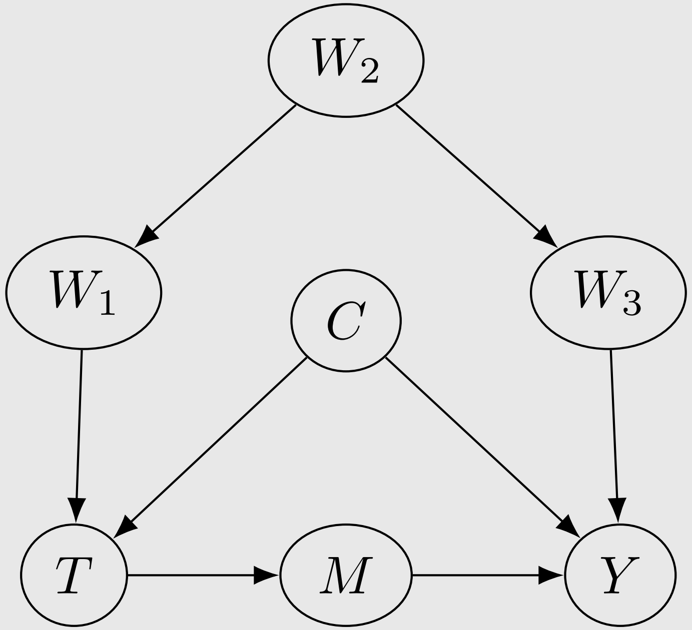
Backdoor adjustment

Structural causal models

A complete example with estimation

Blocking backdoor paths

$P(Y \mid t)$



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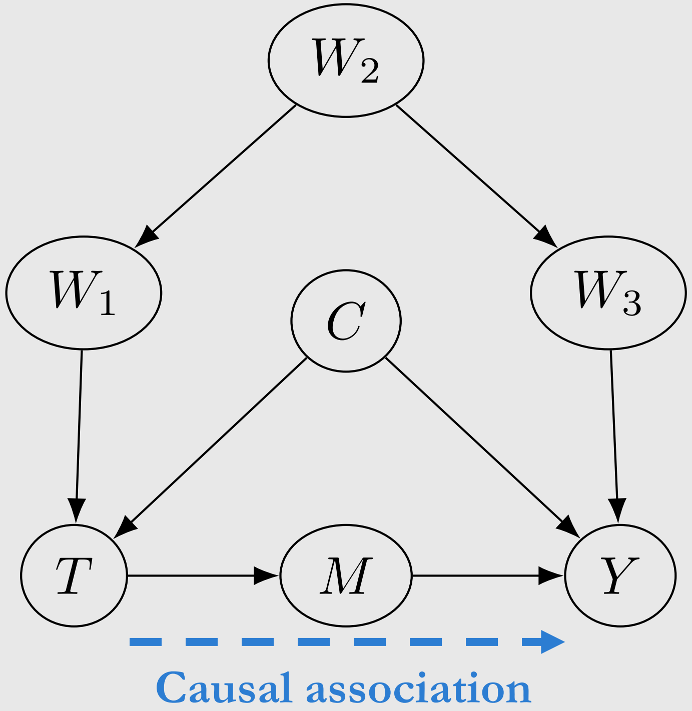

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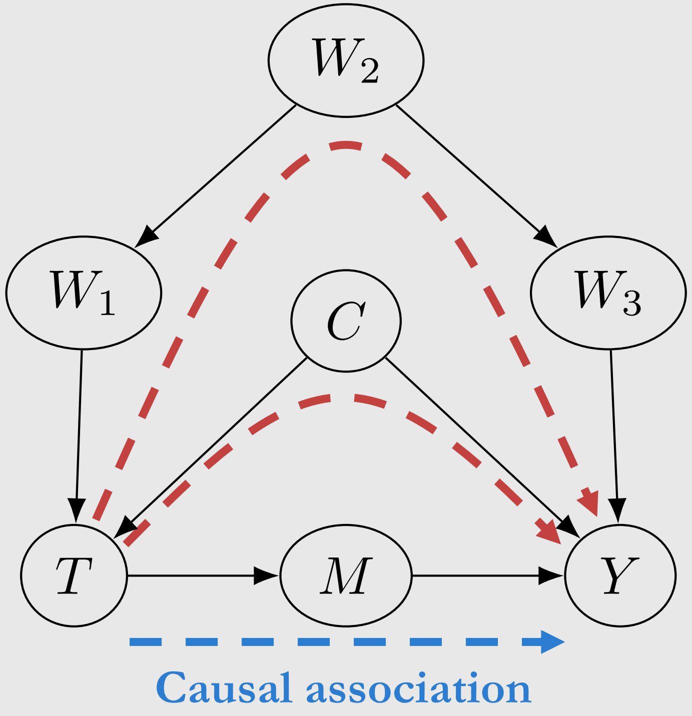

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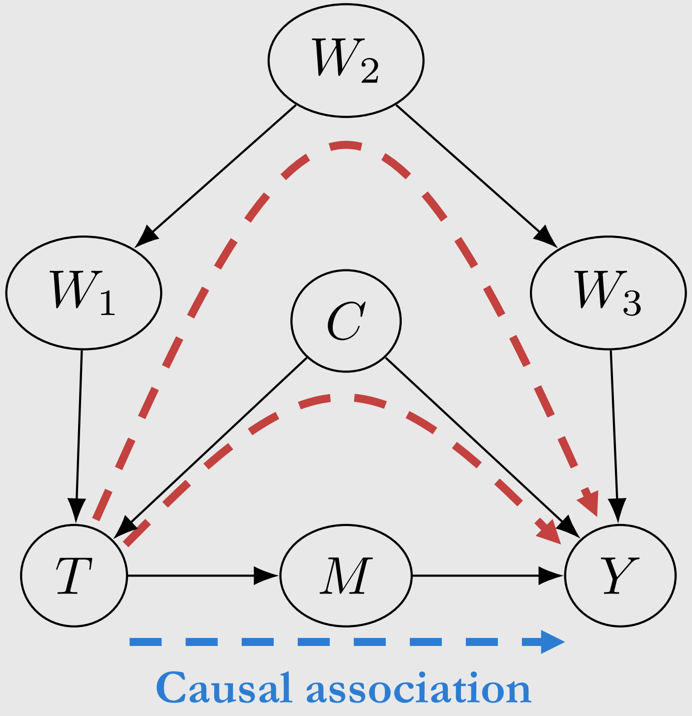

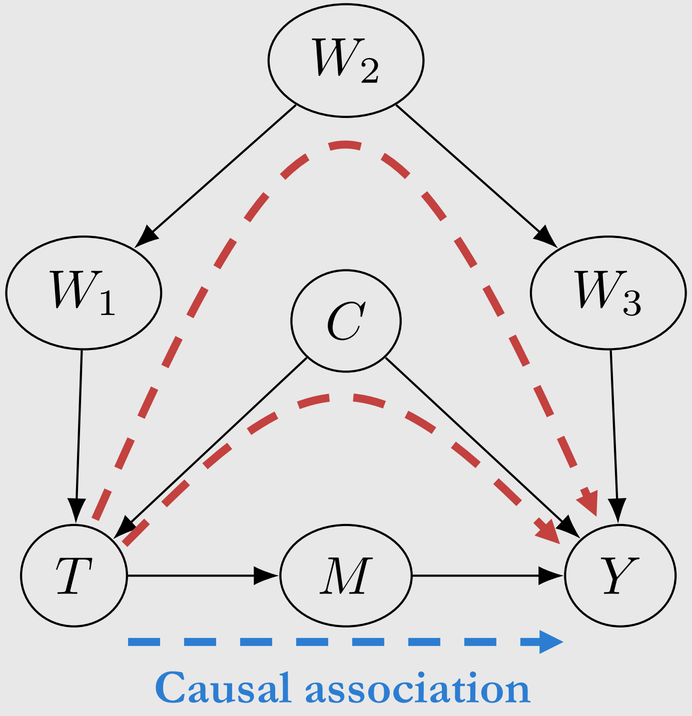

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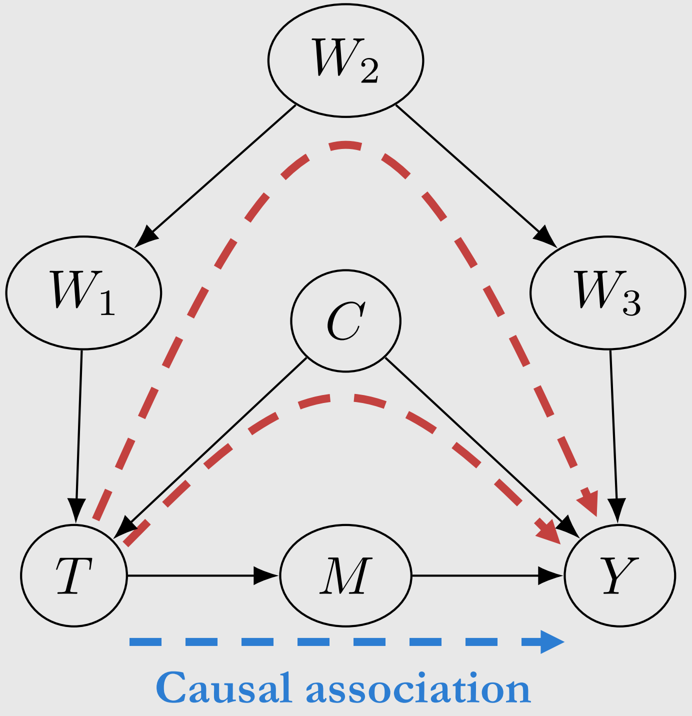

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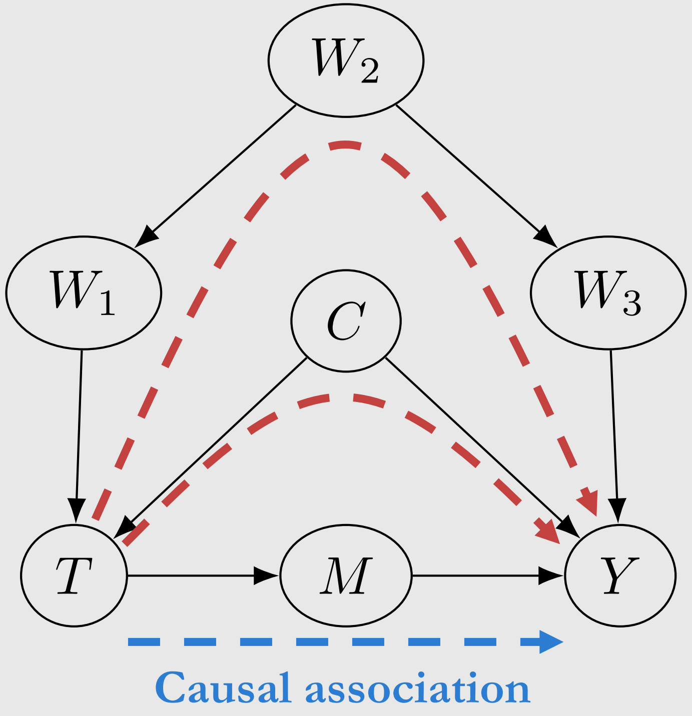

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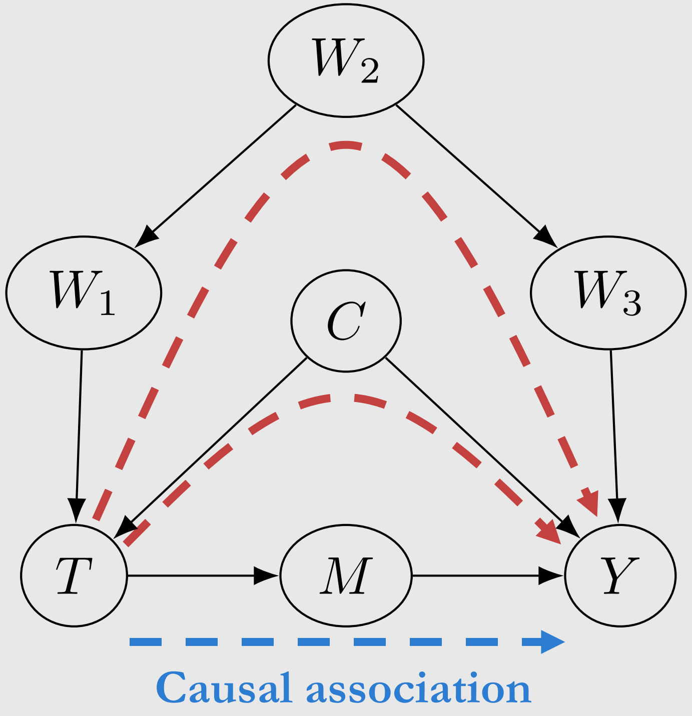

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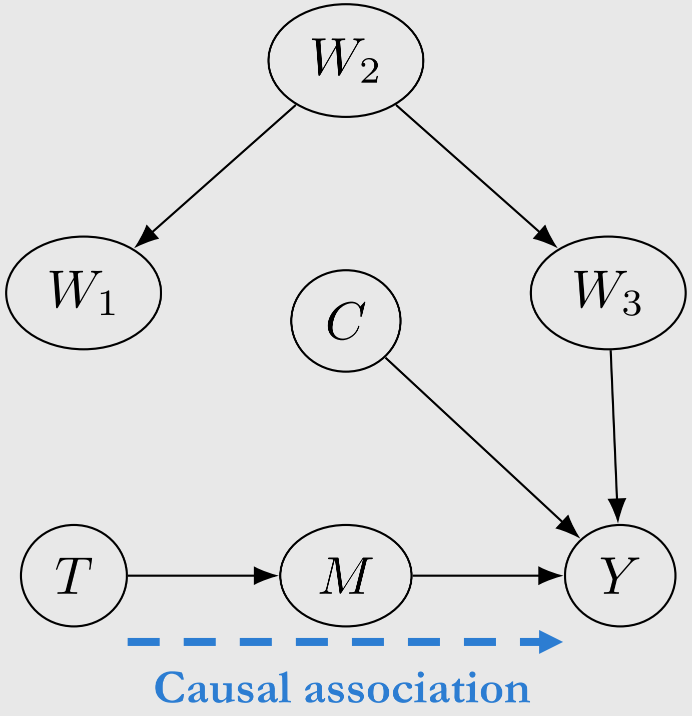
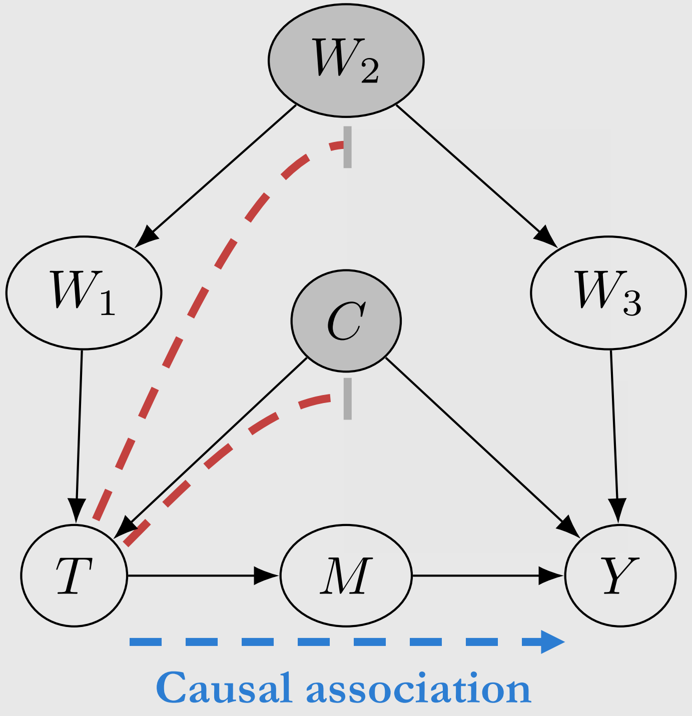


Blocking backdoor paths

$P(Y \mid t, c, w_2)$



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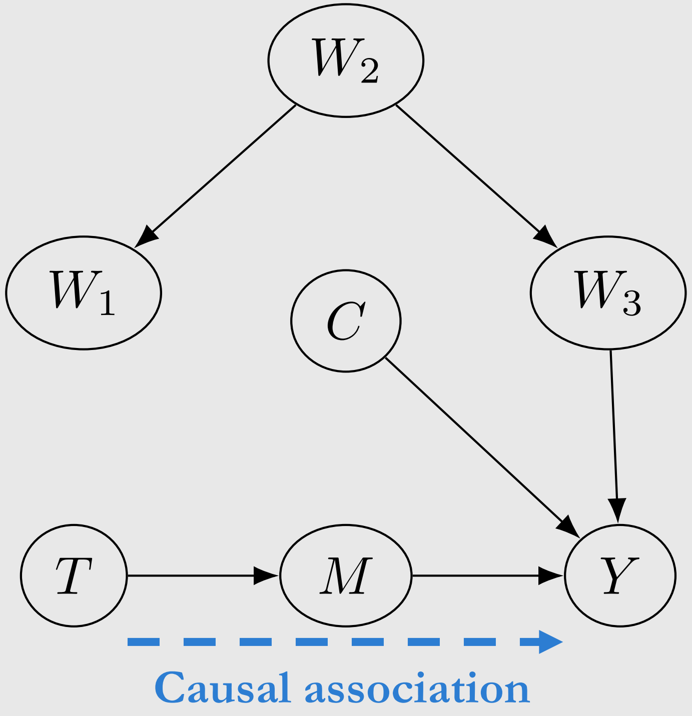
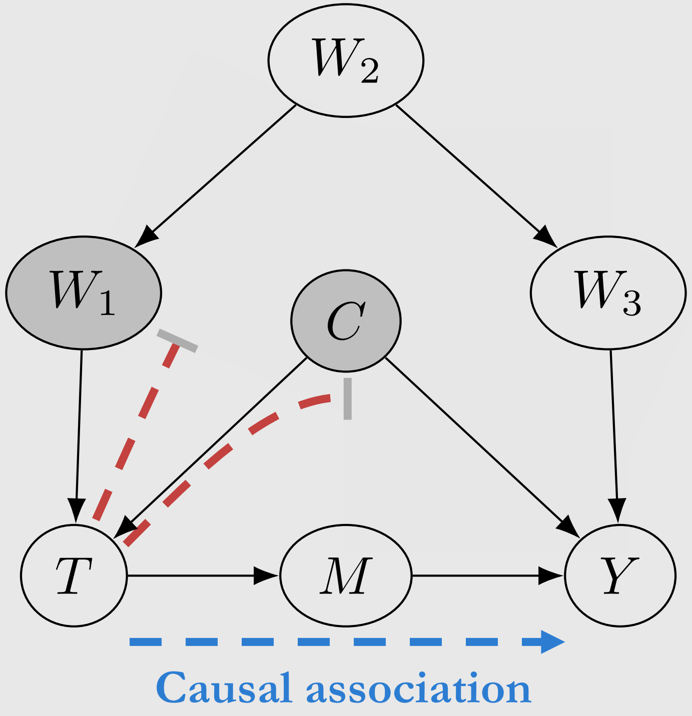
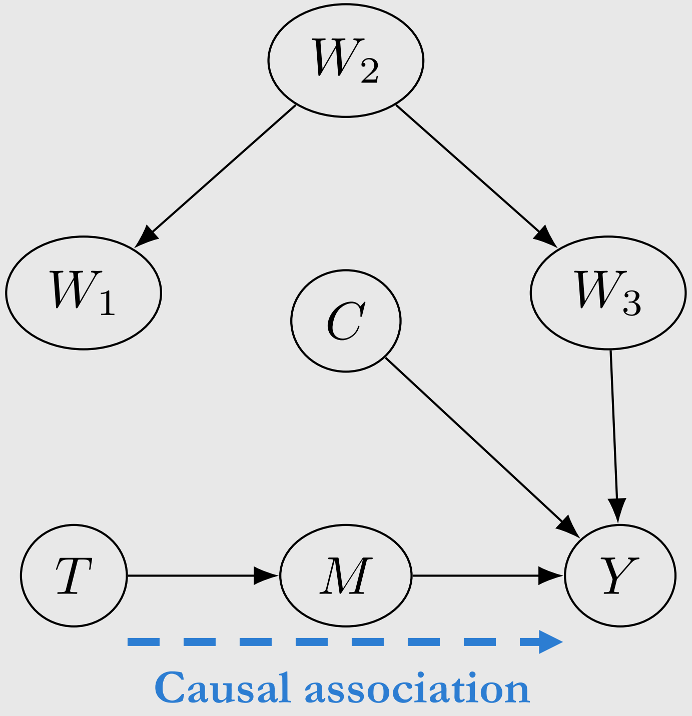


Blocking backdoor paths

$$P(Y \mid t, c, w_1)$$

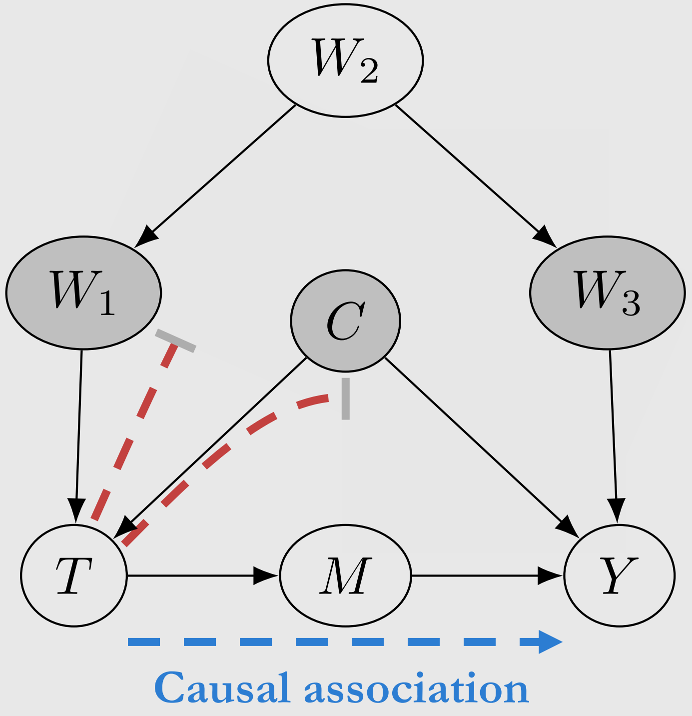


$$P(Y \mid do(t))$$



Blocking backdoor paths

$P(Y \mid t, c, w_1, w_3)$



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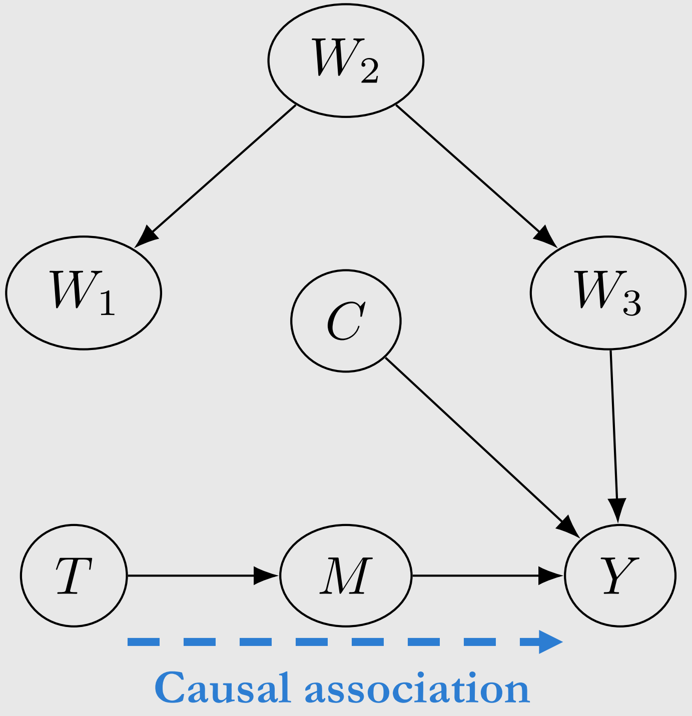

Backdoor criterion and backdoor adjustment

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Given the modularity assumption and that W satisfies the backdoor criterion, we can identify the causal effect of T on Y:

$$P(y \mid do(t)) = \sum_{w} P(y \mid t, w)\, P(w)$$

Proof of backdoor adjustment

$$P(y \mid do(t))$$

$$= \sum_w P(y \mid t, w) \, P(w)$$

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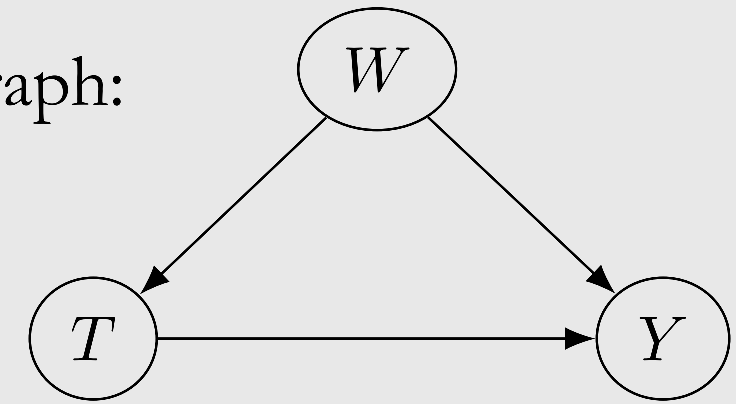
Example graph:



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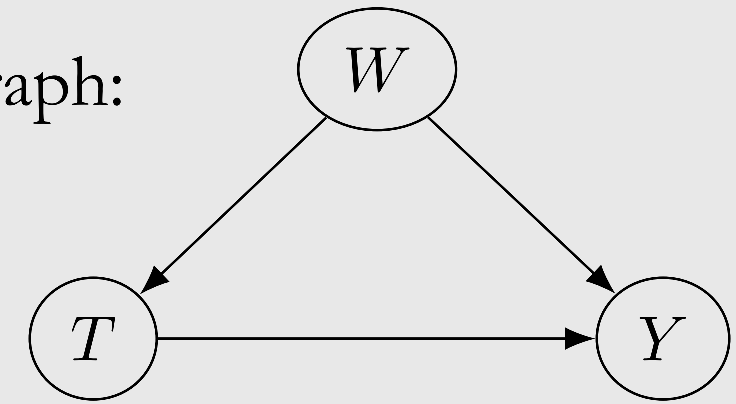

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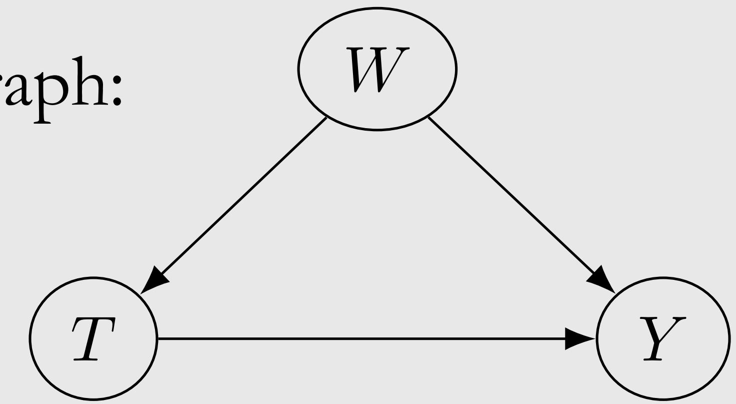

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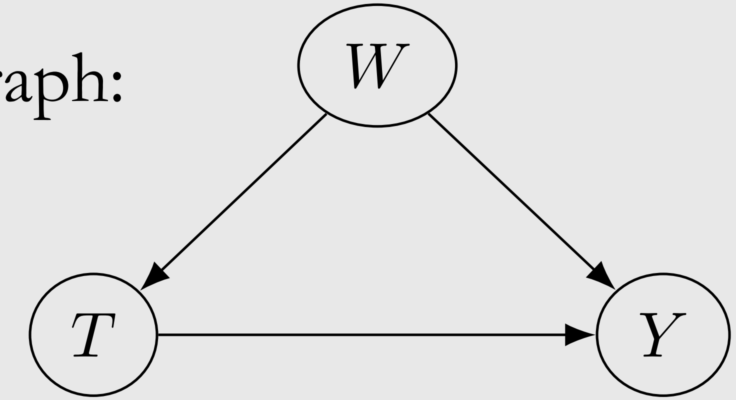
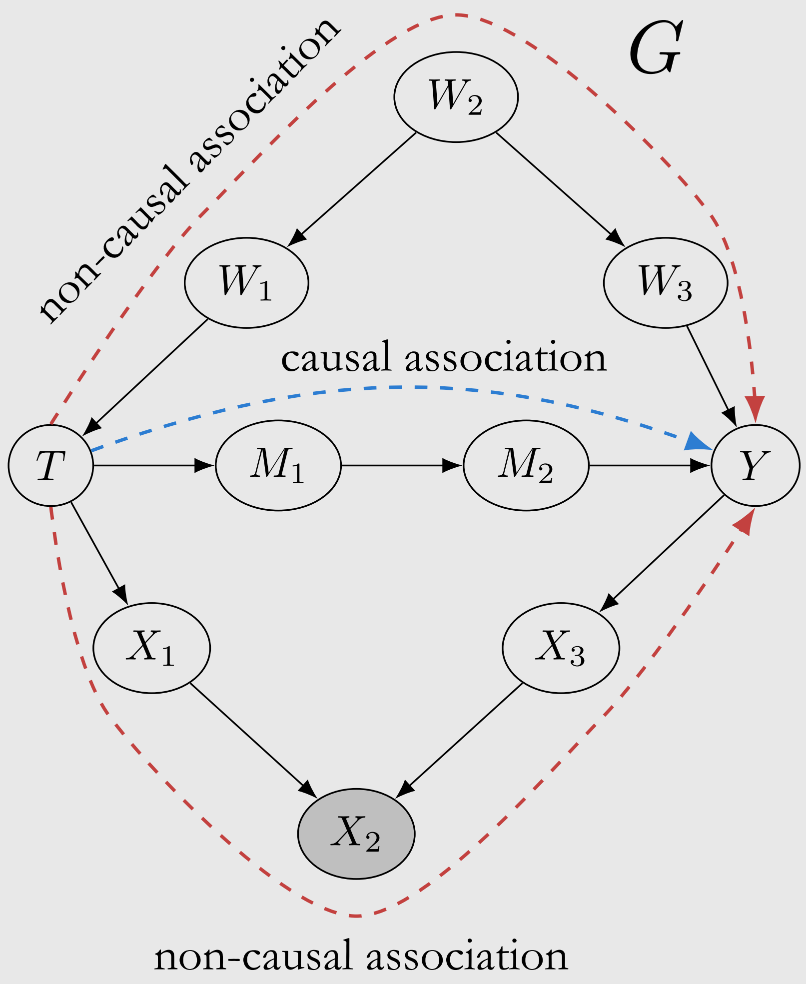


Backdoor criterion as d-separation



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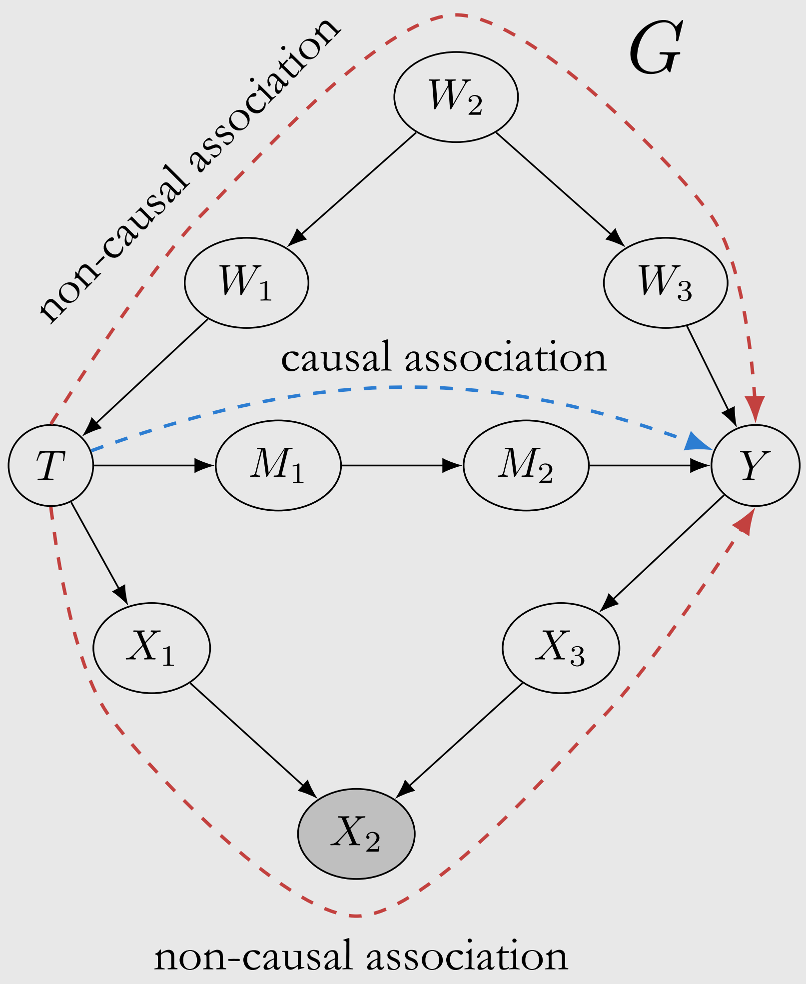

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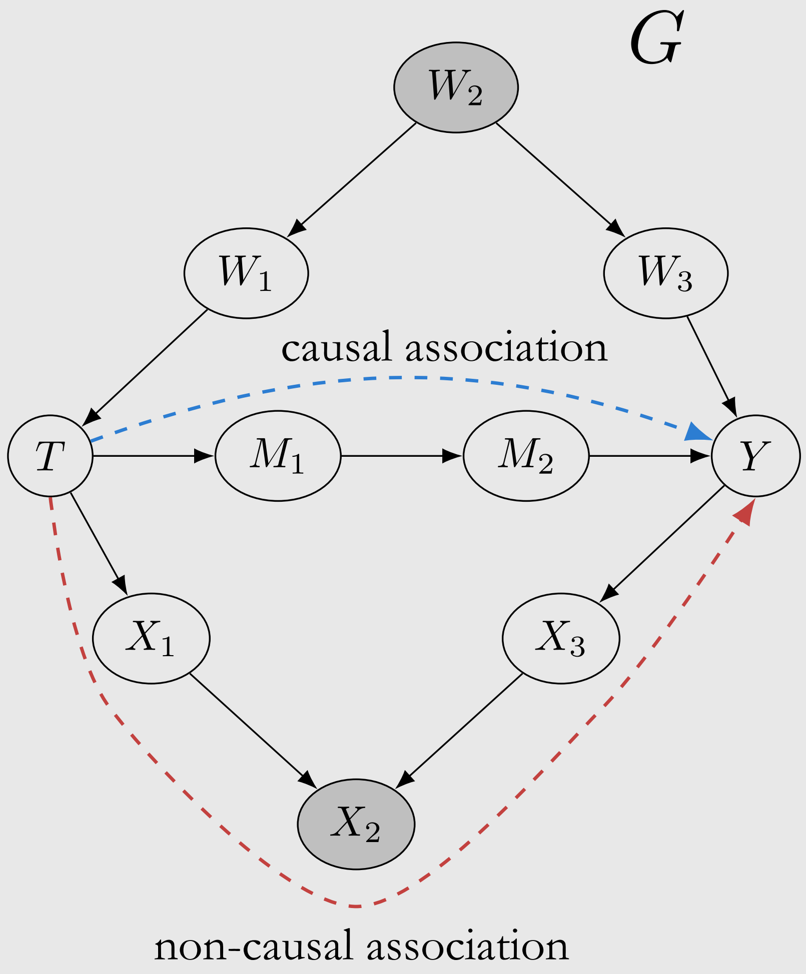

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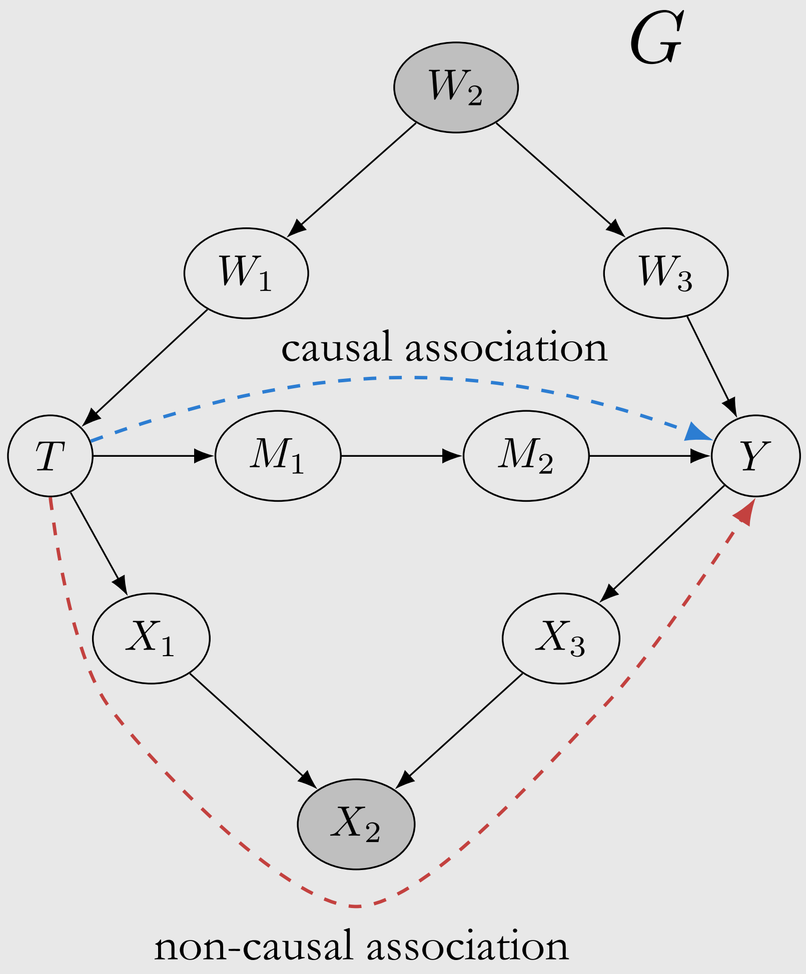

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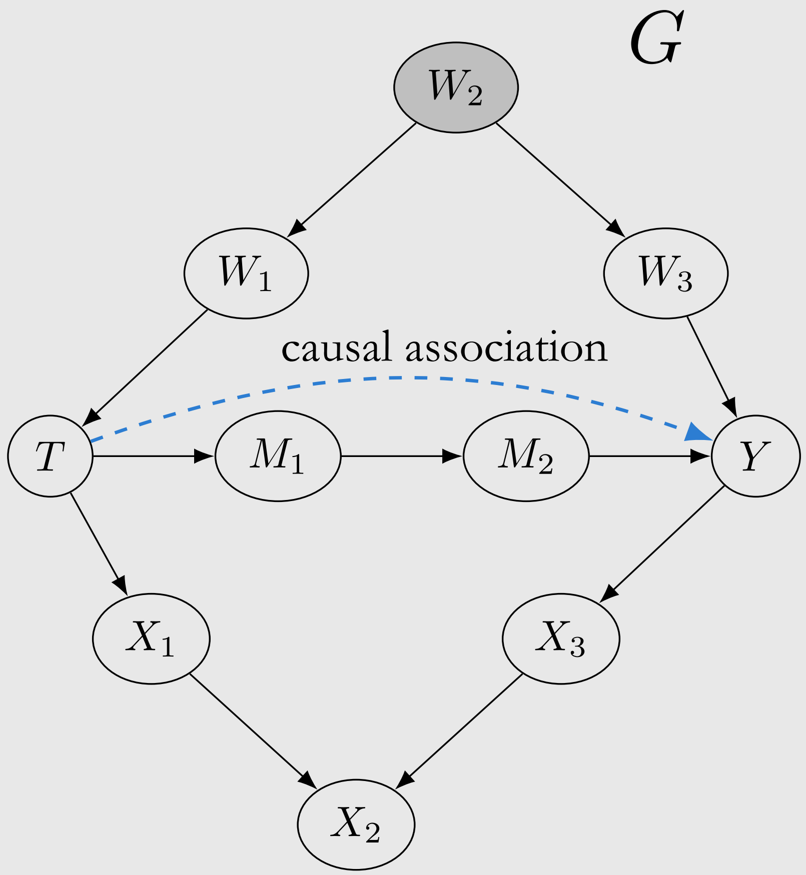

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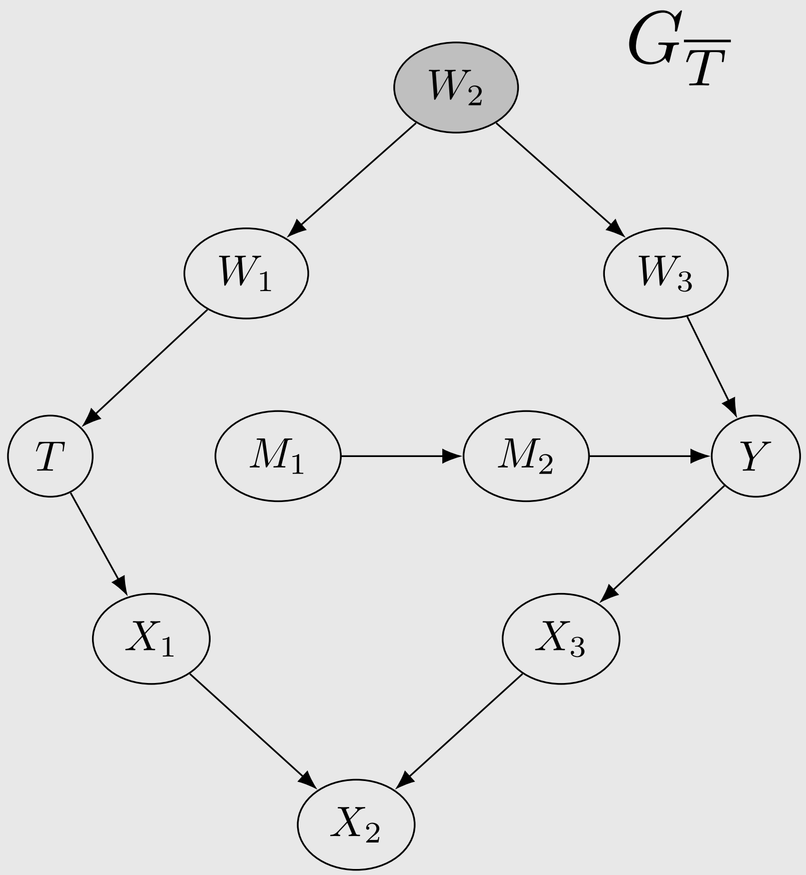

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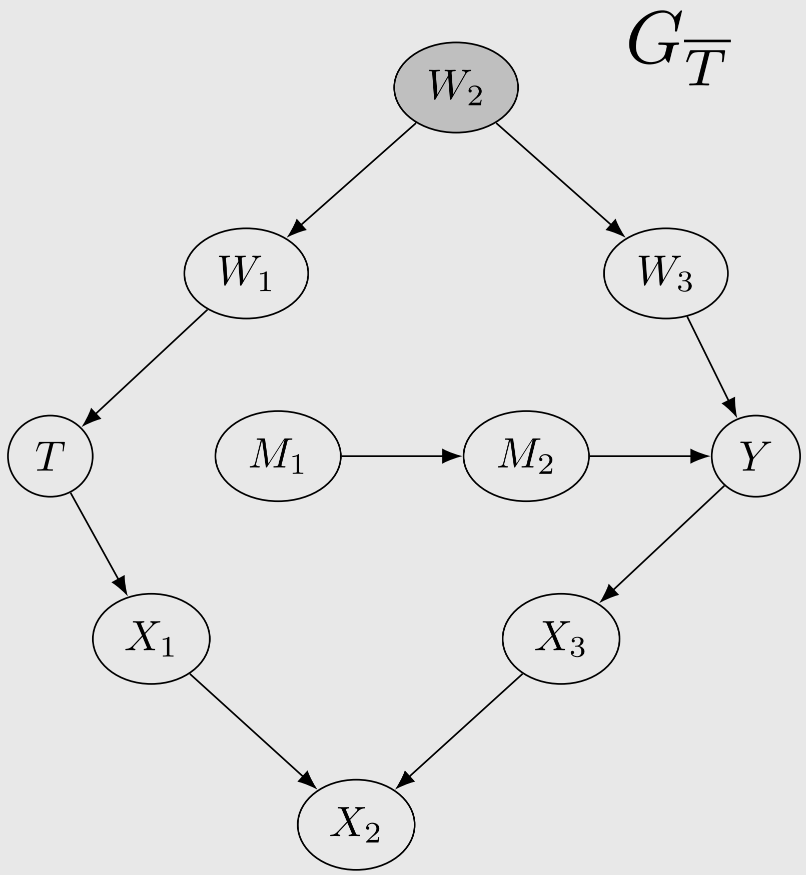

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$$Y \perp\!\!\!\perp_{G_{\overline{T}}} T \mid W$$

Question:
How does this backdoor adjustment relate to the adjustment formula we saw in the potential outcomes lecture?

Backdoor adjustment:

$$P(y \mid do(t)) = \sum_w P(y \mid t, w)\, P(w)$$

Adjustment formula from before:

$$\mathbb{E}[Y(1) - Y(0)] = \mathbb{E}_W \left[\mathbb{E}[Y \mid T=1, W] - \mathbb{E}[Y \mid T=0, W]\right]$$

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Section 4.4.1 of the ICI book

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The *do*-operator

Main assumption: modularity

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Structural causal models

A complete example with estimation

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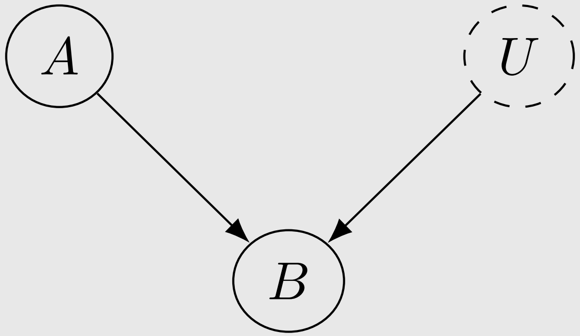
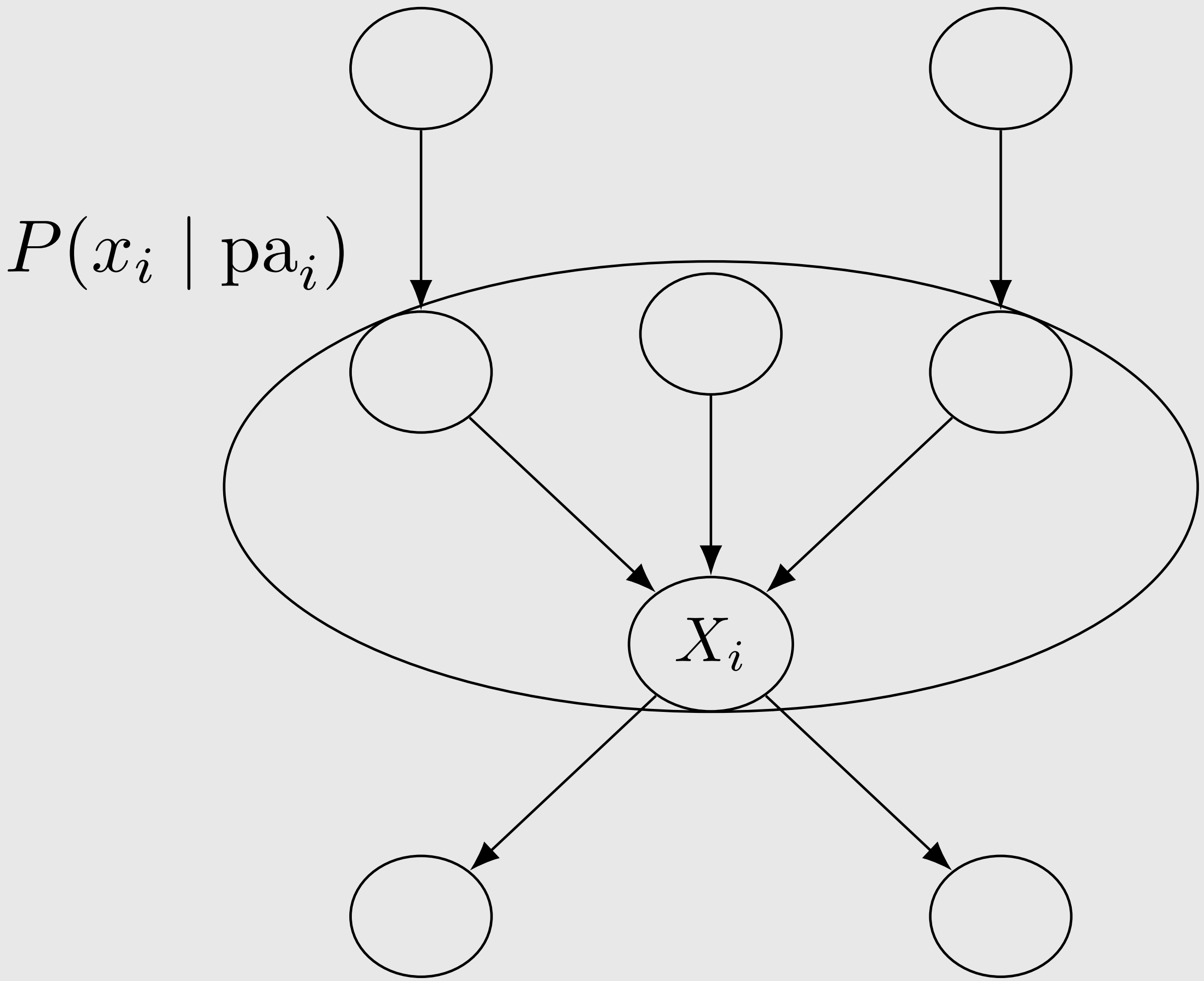


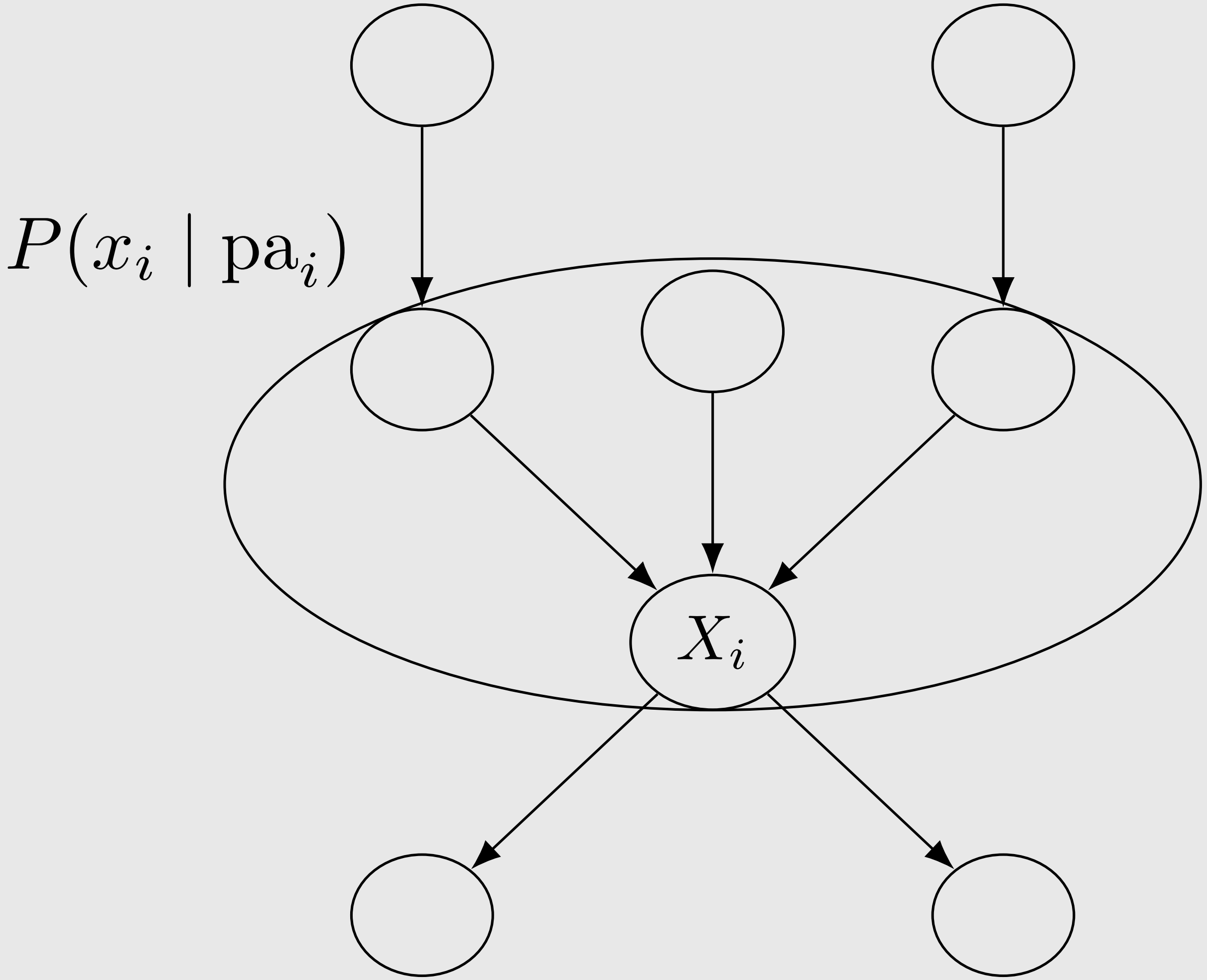
Causal mechanisms and direct causes revisited



Causal mechanisms and direct causes revisited

Causal mechanism for X_i

$$X_i := f(A, B, \ldots)$$



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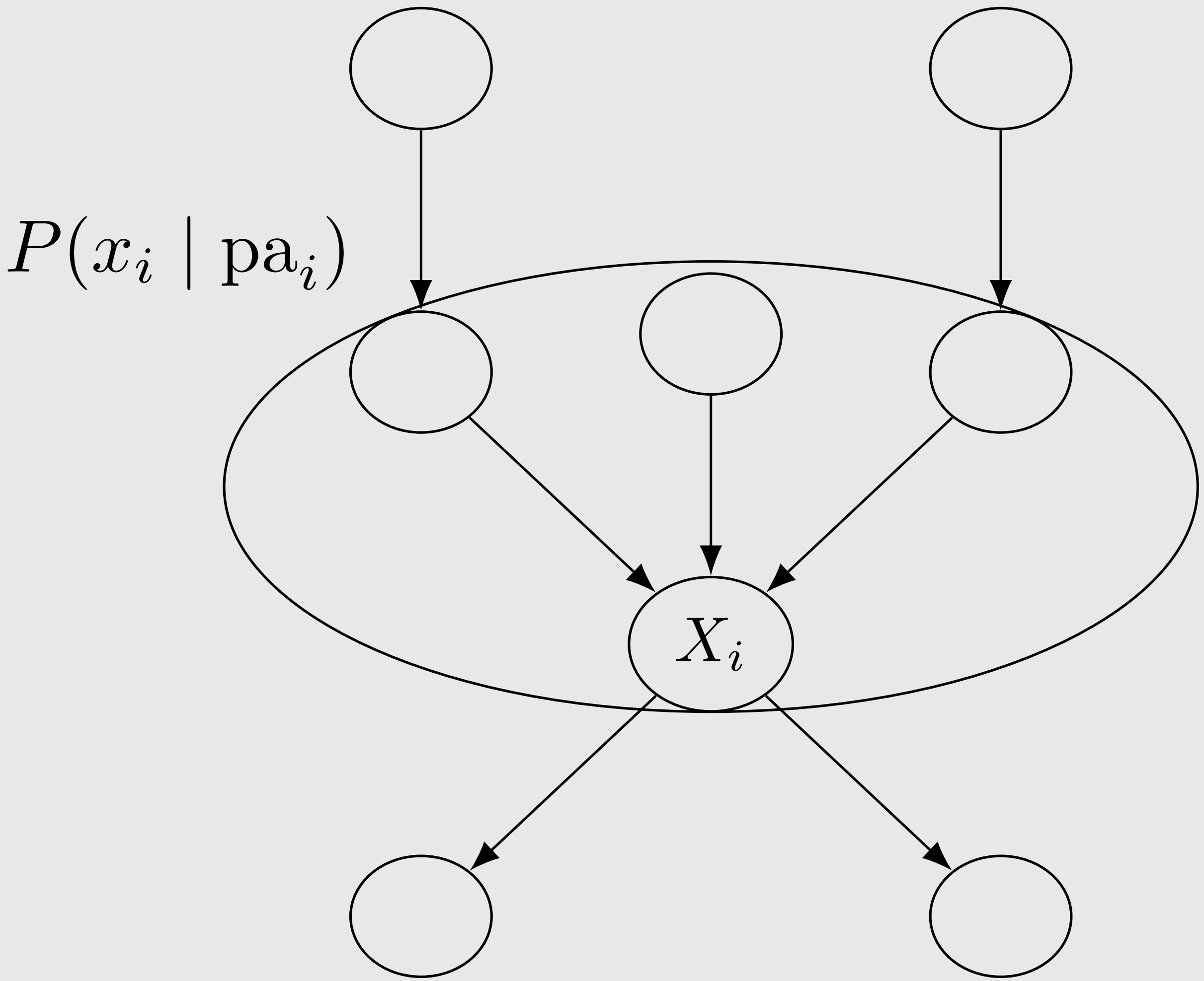
Direct causes of X_i



Structural causal models (SCMs)

$$M: \quad \begin{aligned} B &:= f_B(A, U_B) \\ C &:= f_C(A, B, U_C) \\ D &:= f_D(A, C, U_D) \end{aligned}$$

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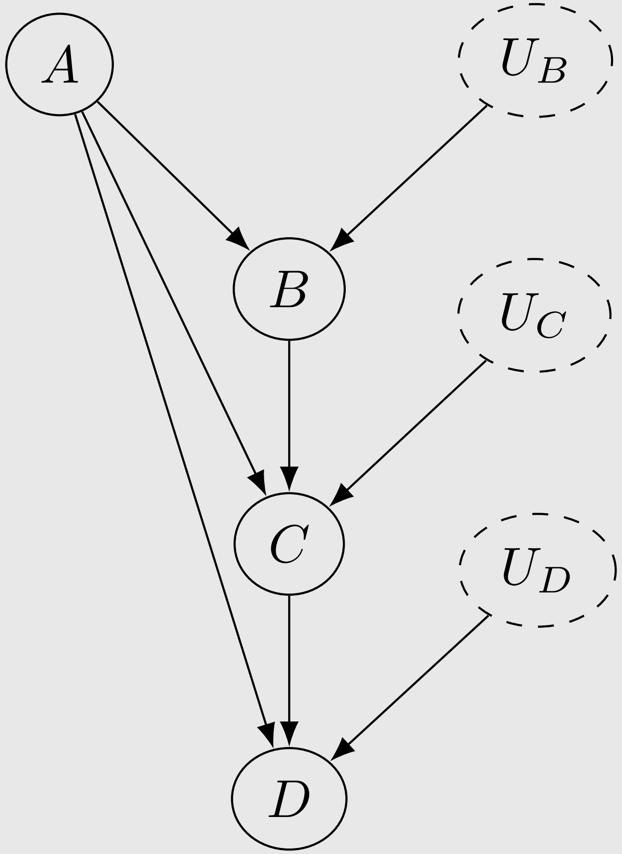

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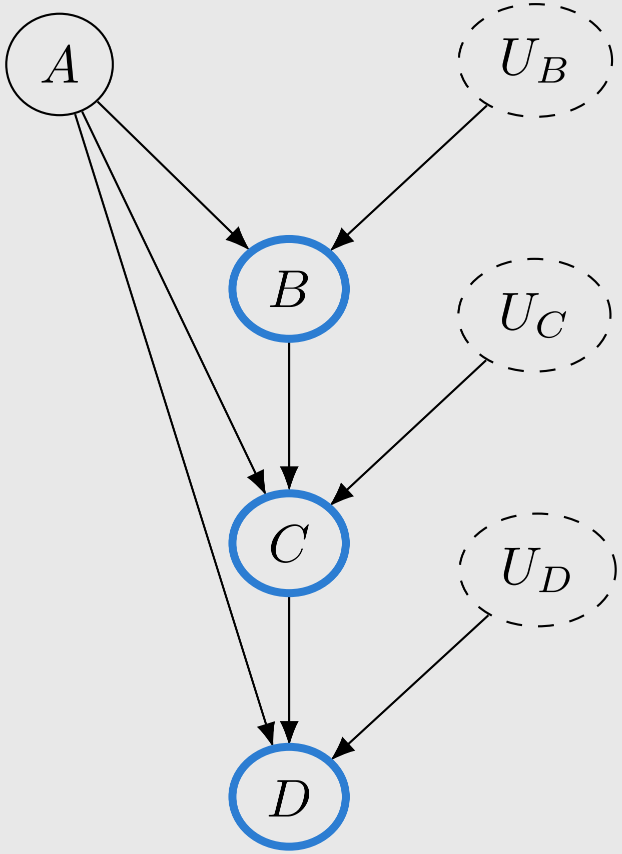

Endogenous variables

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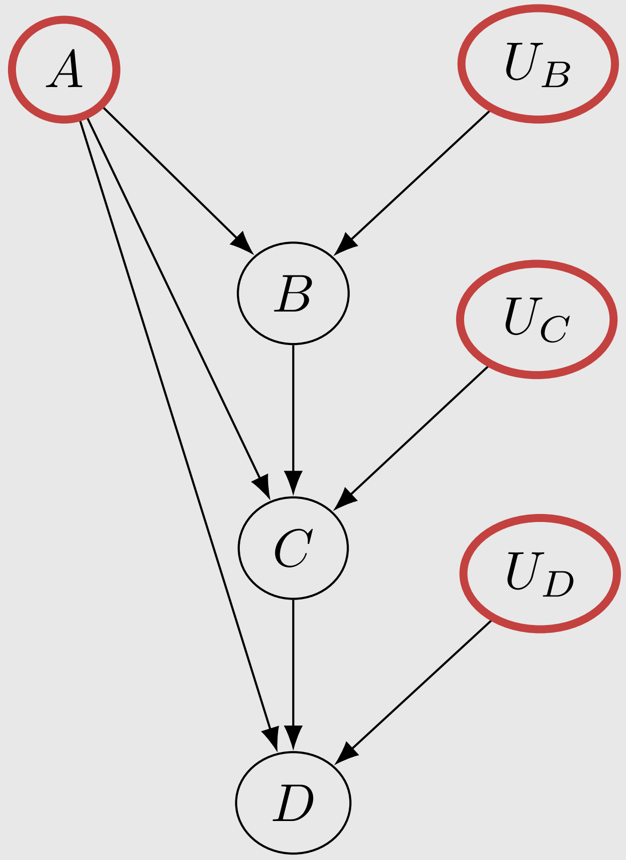

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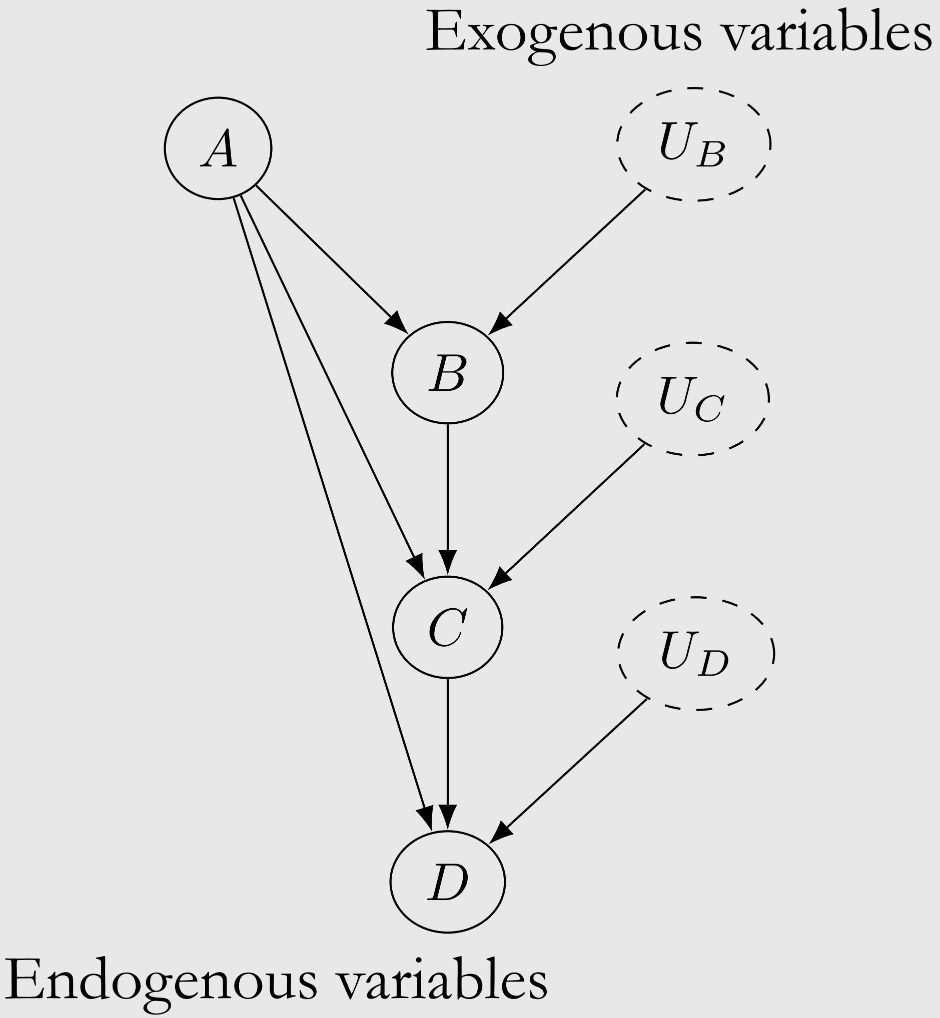


SCM Definition

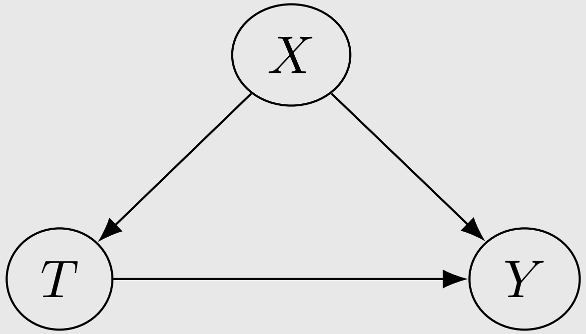
A tuple of the following sets:

1. A set of endogenous variables
2. A set of exogenous variables
3. A set of functions, one to generate each endogenous variable as a function of the other variables

Interventions

SCM
(model)

$$M: \quad \begin{aligned} T &:= f_T(X, U_T) \\ Y &:= f_Y(X, T, U_Y) \end{aligned}$$



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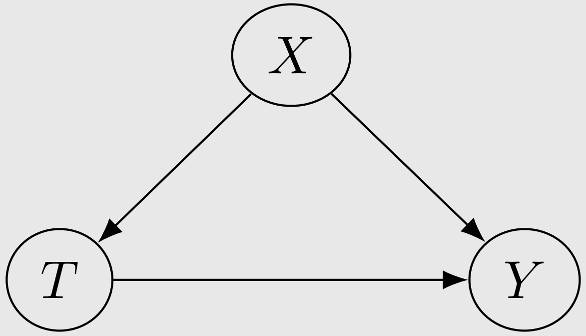

Interventional SCM
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$$M_t: \quad \begin{aligned} T &:= t \\ Y &:= f_Y(X, T, U_Y) \end{aligned}$$

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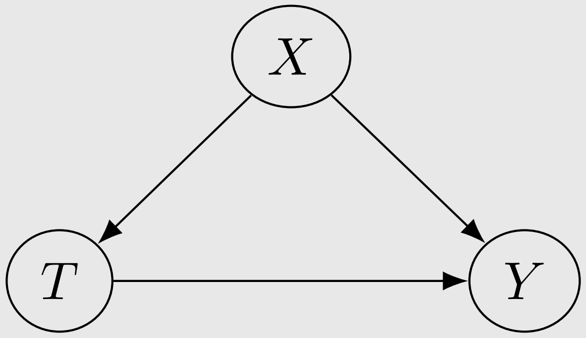

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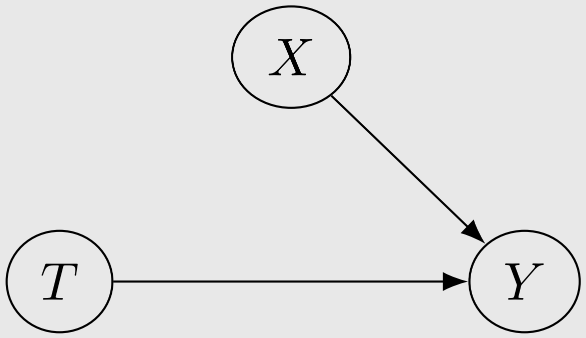

Modularity assumption for SCMs

Consider an SCM M and an interventional SCM M_t that we get by performing the intervention $do(T = t)$. The modularity assumption states that M and M_t share all of their structural equations except the structural equation for T, which is $T := t$ in M_t.

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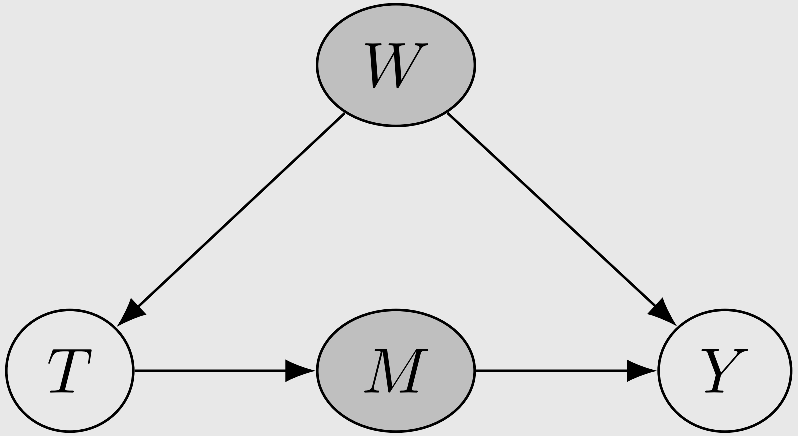
Why not condition on descendants of treatment: blocking causal association



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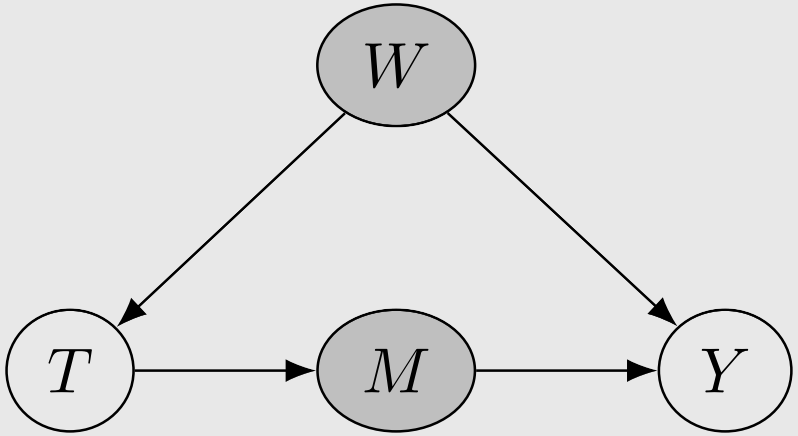
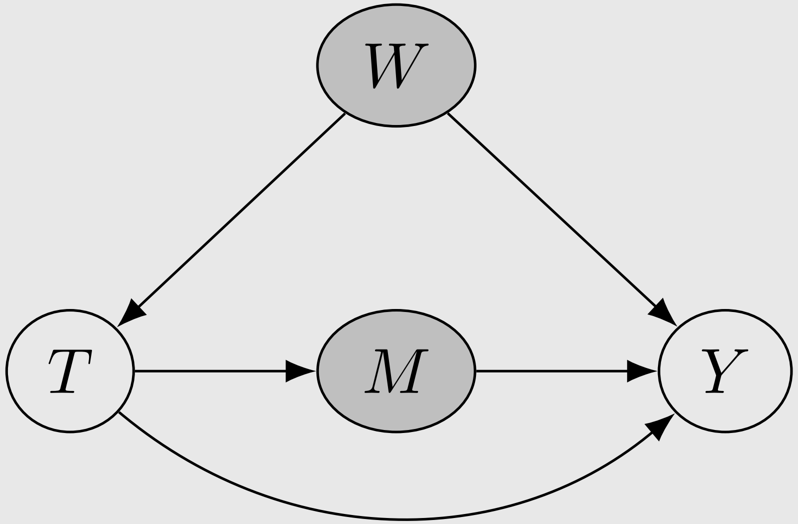
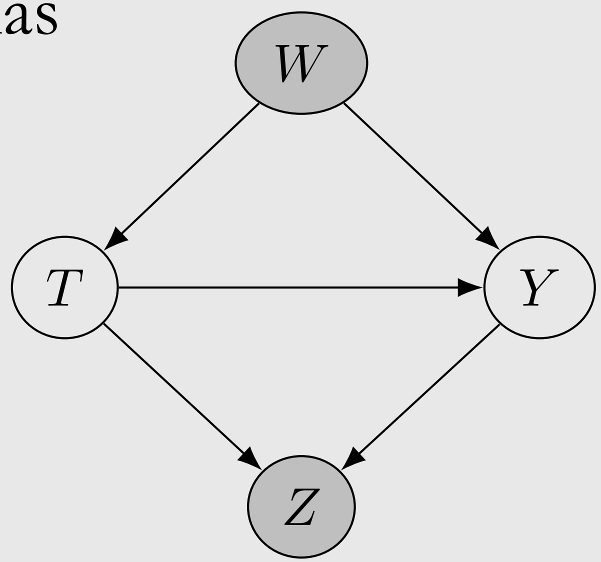




Why not condition on descendants of treatment: inducing new post-treatment association

Collider bias



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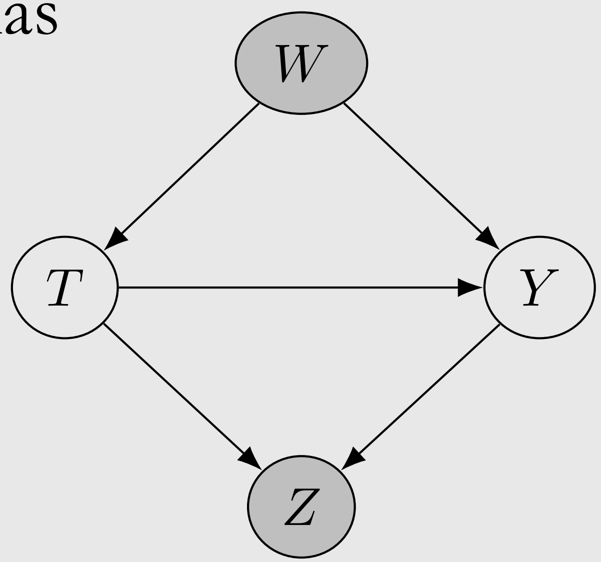

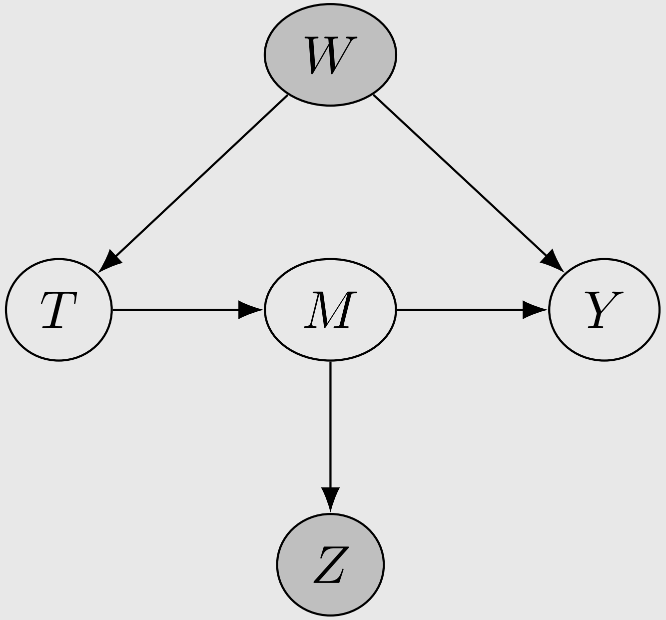

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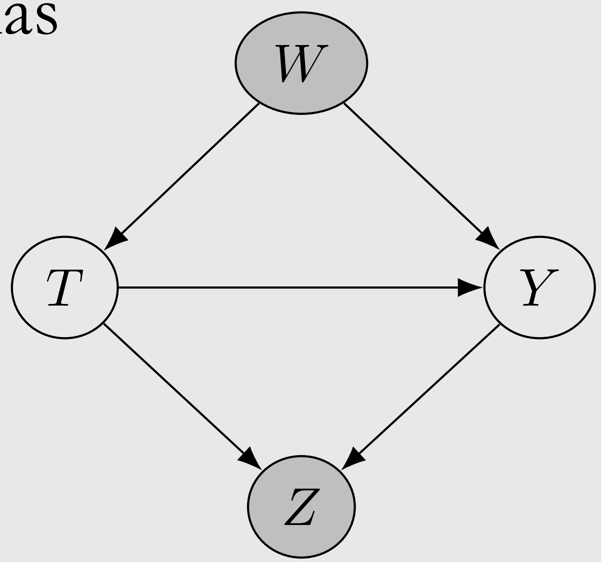

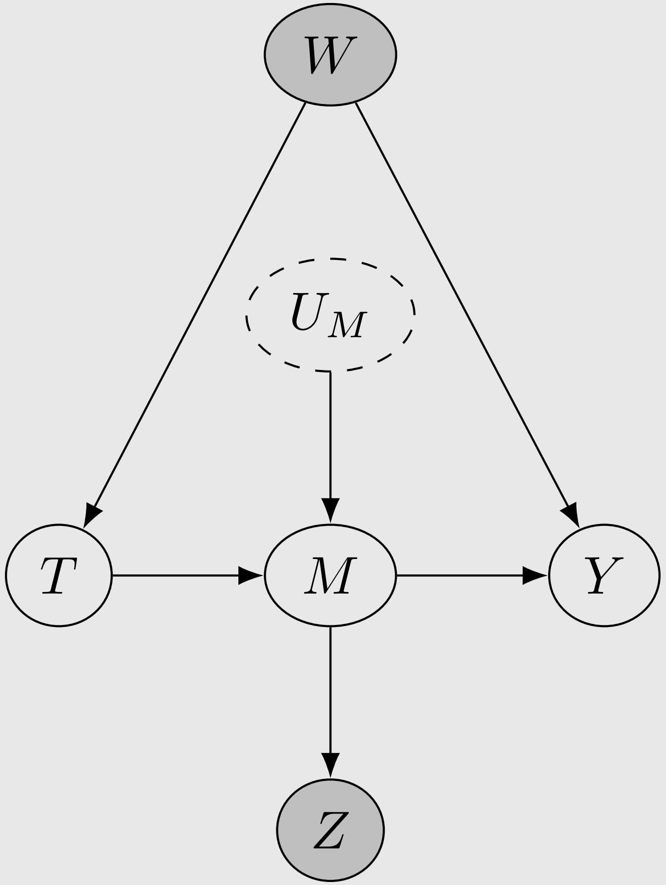

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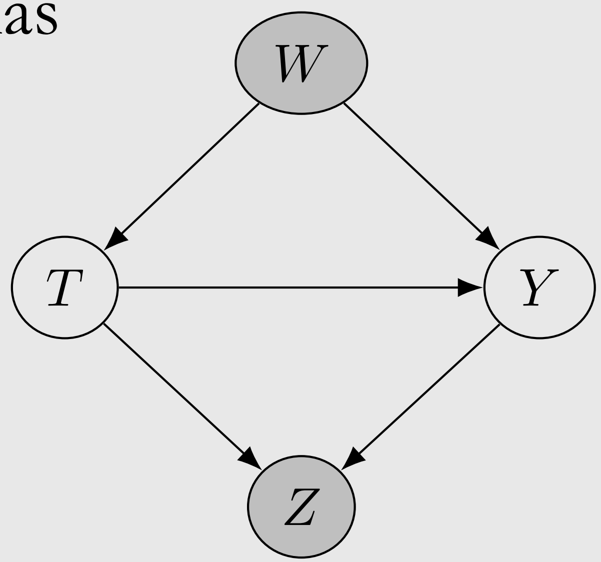
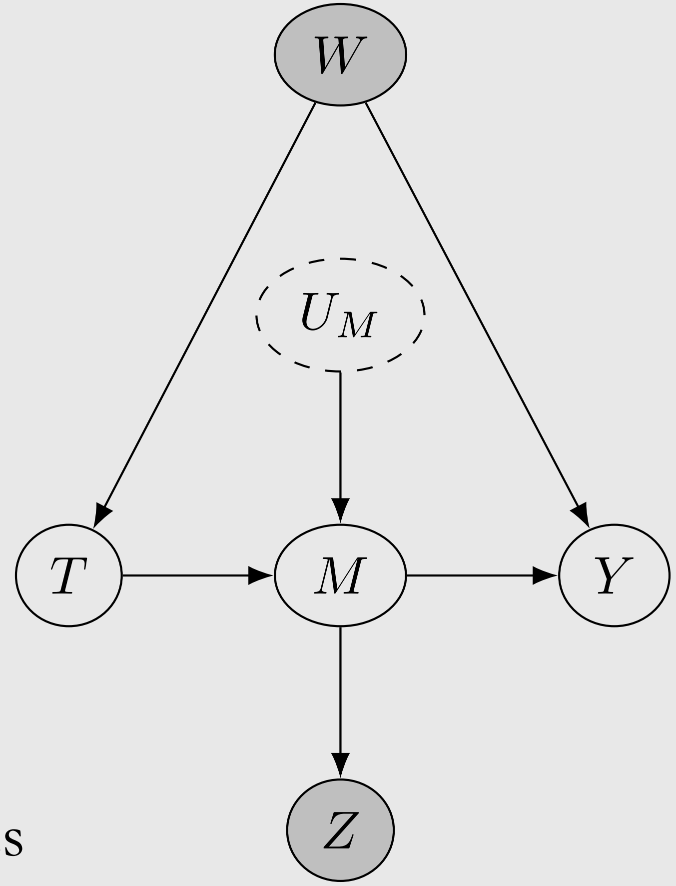
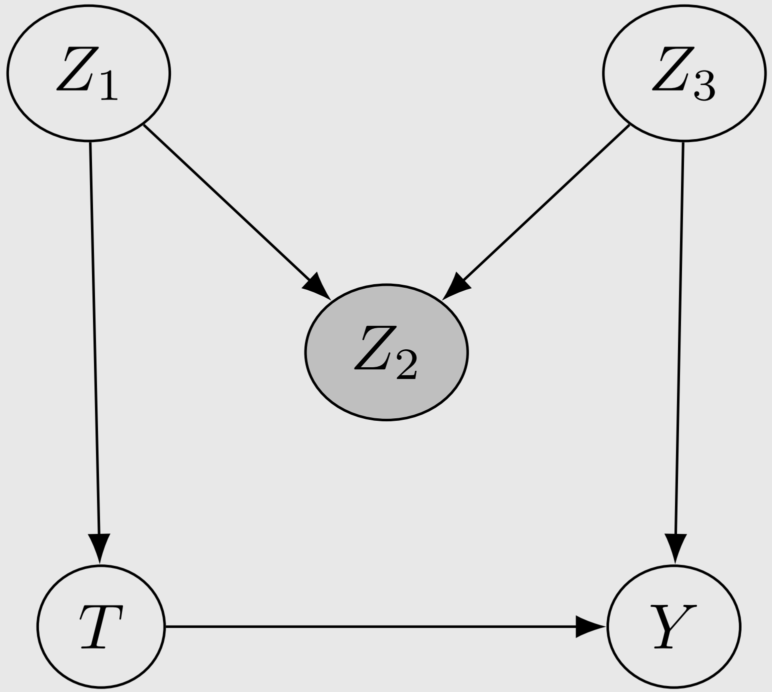


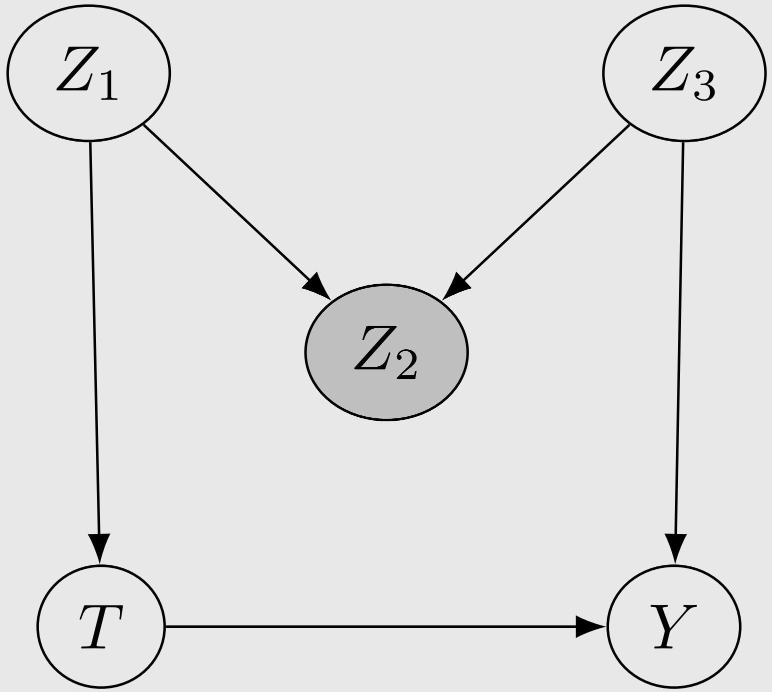


Rule: don't condition on post-treatment covariates

Inducing new **pre**treatment association (M-bias)



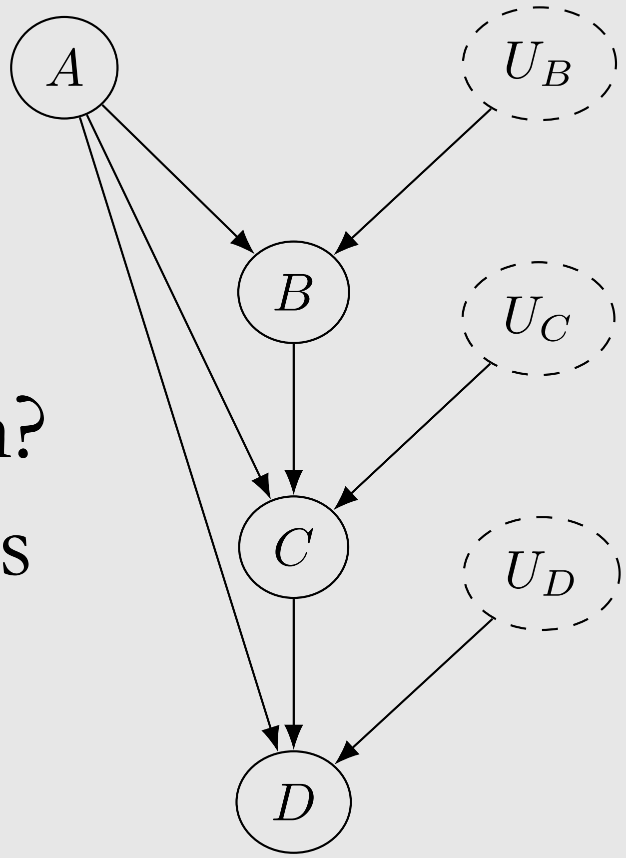
Inducing new **pre**treatment association (M-bias)



See Elwert & Winship (2014) for many real examples of collider bias

Questions:

1. What are are the nonparametric structural equations for this causal graph?
2. What are the endogenous and exogenous variables in this causal graph?
3. What is collider bias?



The *do*-operator

Main assumption: modularity

Backdoor adjustment

Structural causal models

A complete example with estimation

Problem: effect of sodium intake on blood pressure

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Motivation: 46% of Americans have high blood pressure and high blood pressure is associated with increased risk of mortality

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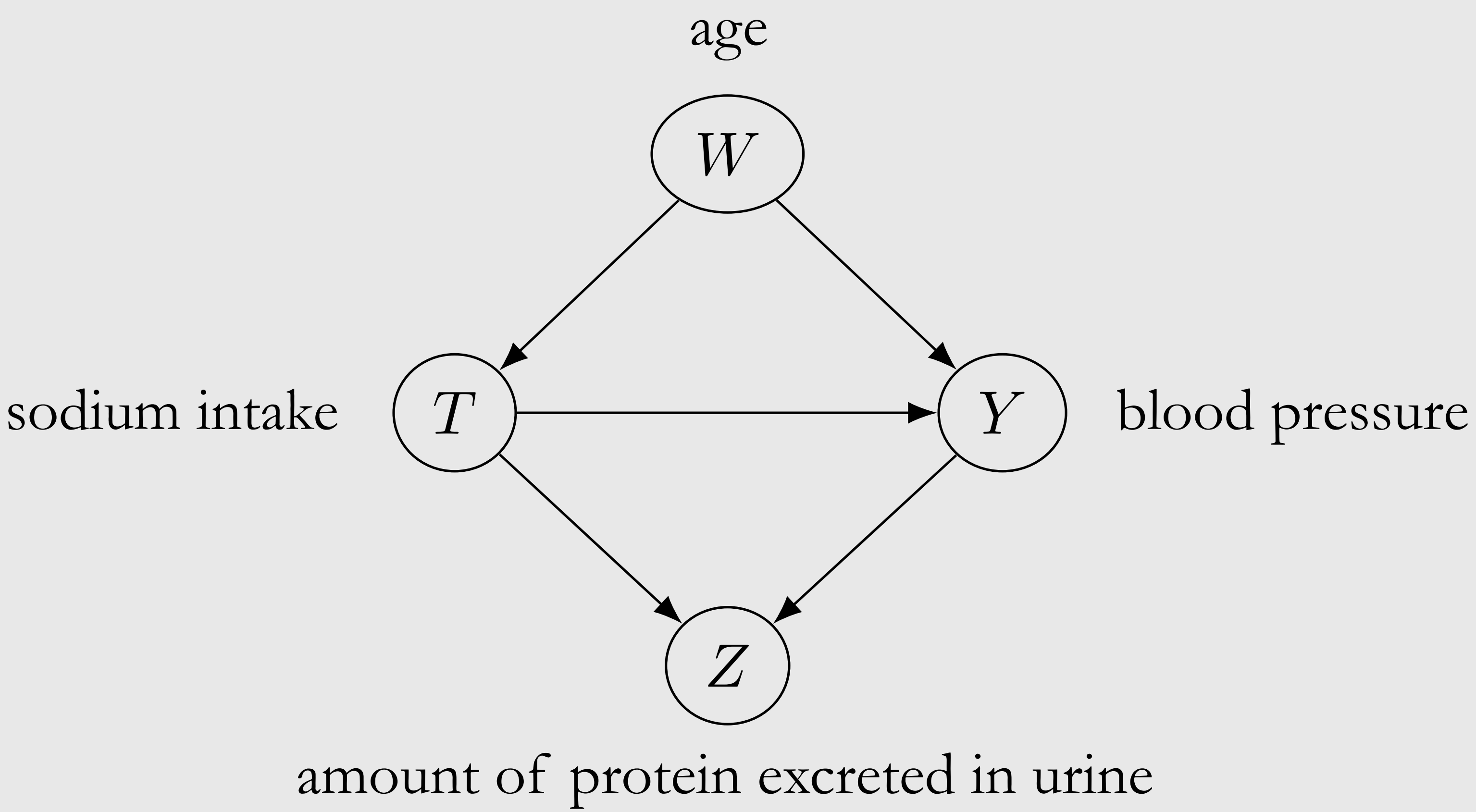
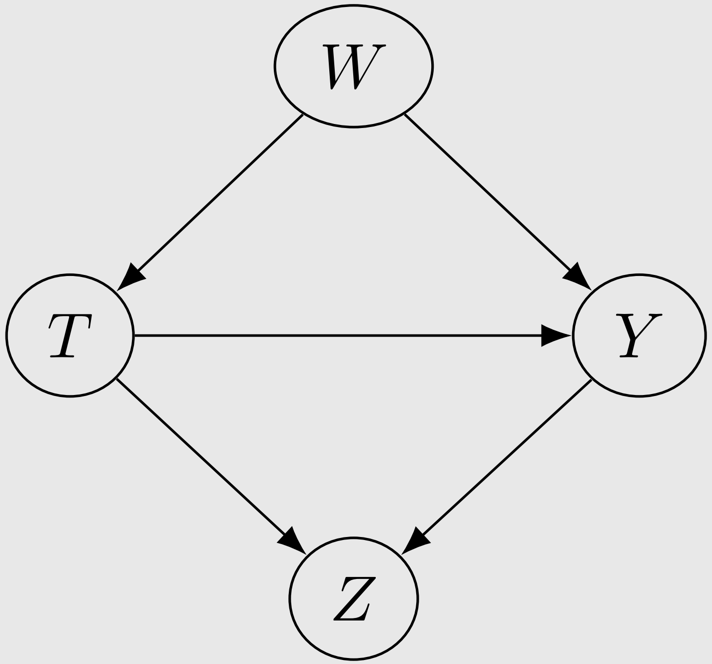
- Simulation: so we know the "true" ATE is 1.05

The causal graph



Identification



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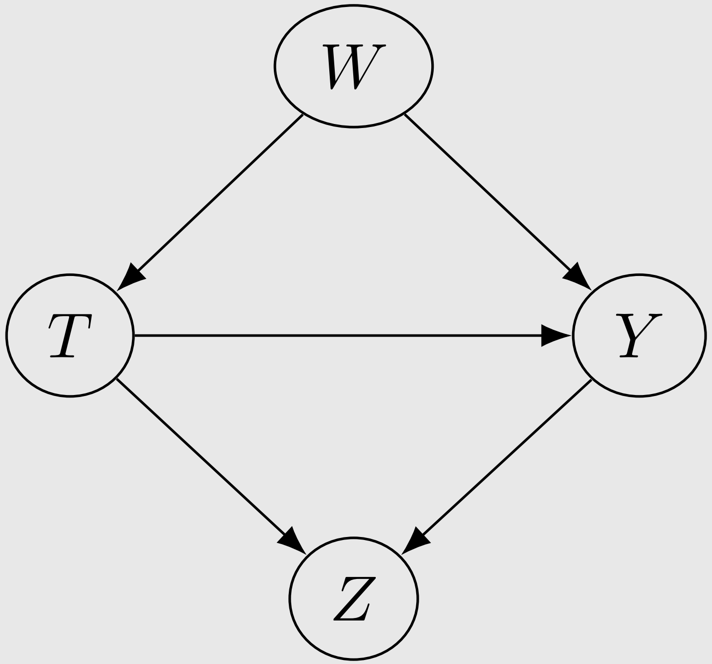

Causal estimand: $\mathbb{E}[Y \mid do(t)]$

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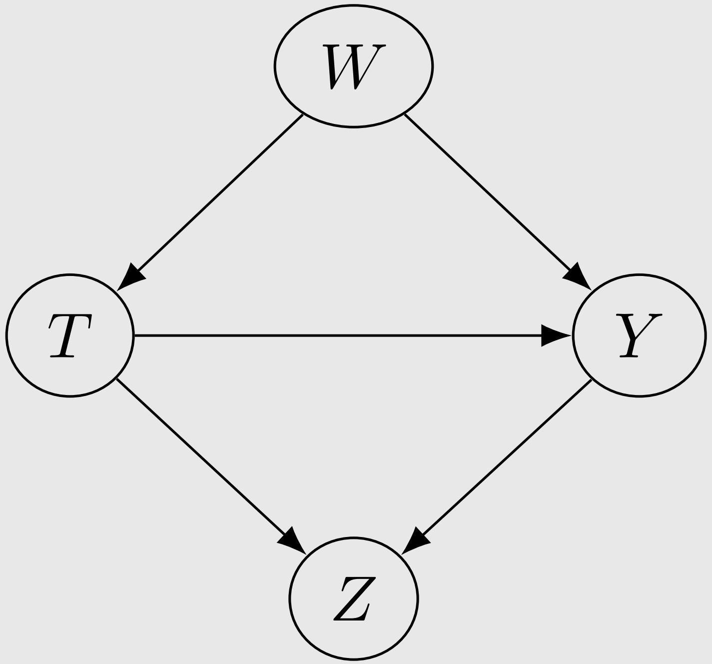

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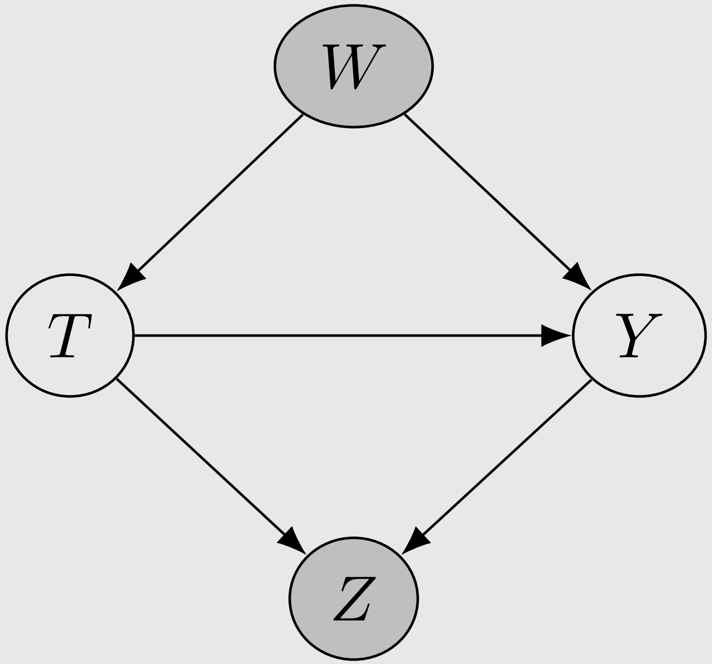
Identification



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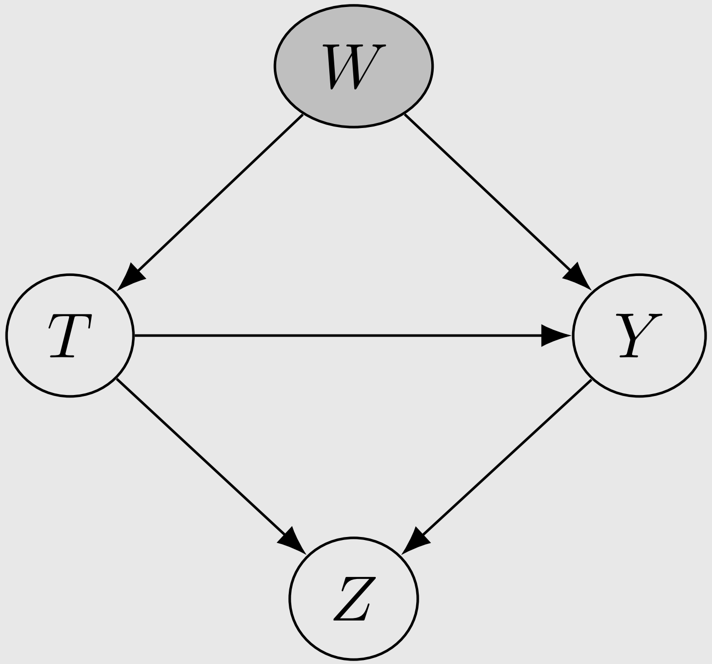

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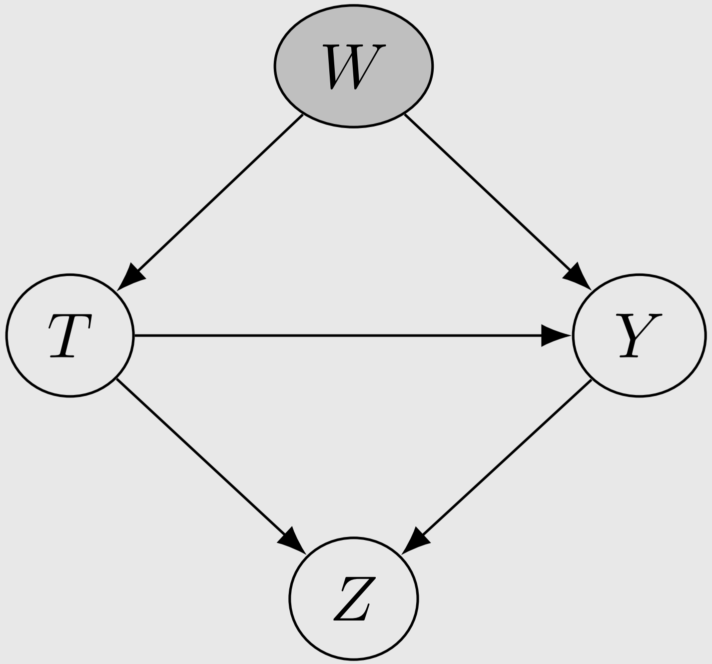

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Statistical estimand from causal graph: $\mathbb{E}_W\mathbb{E}[Y \mid t, W]$

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True ATE: $\mathbb{E}[Y(1) - Y(0)] = 1.05$

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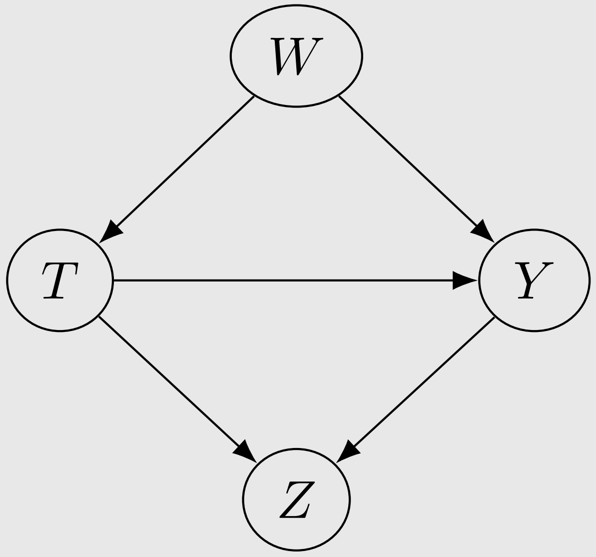

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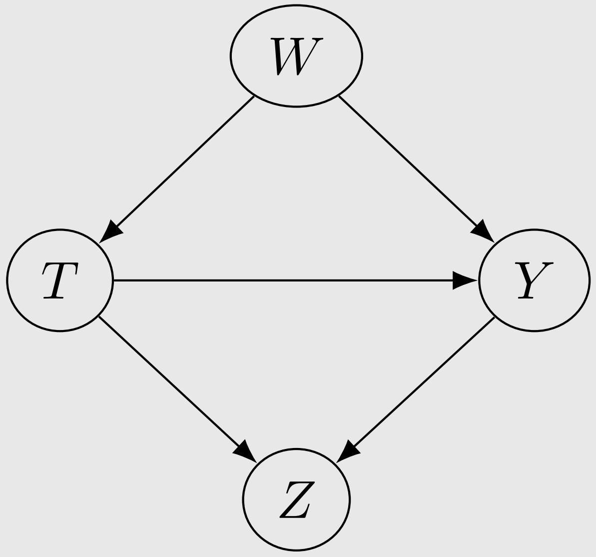

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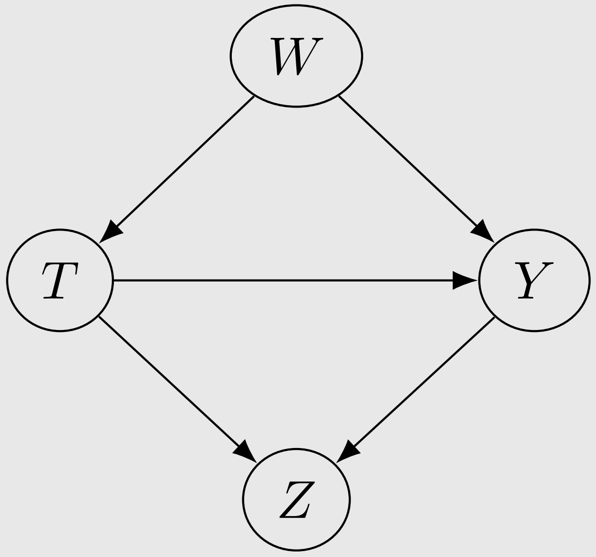
$\frac{|5.33 - 1.05|}{1.05} \times 100\% = 407\%$ error



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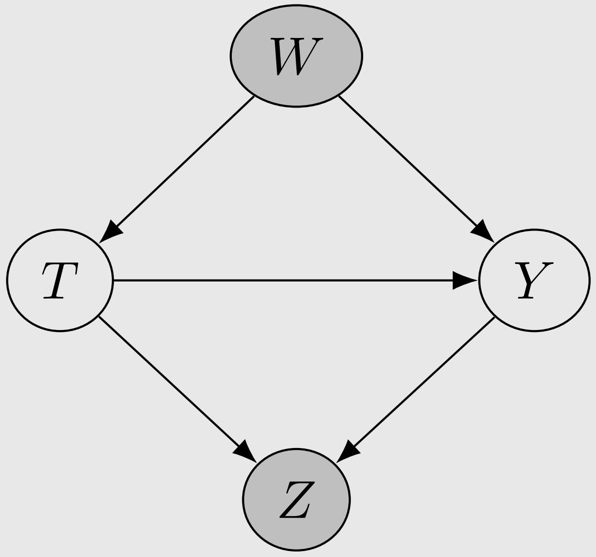
$X = \{W, Z\}$ (last week): 0.85



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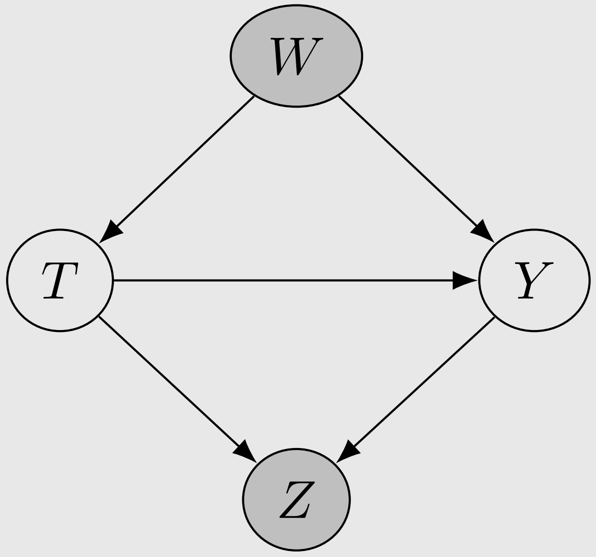
Model (linear regression)

Estimates:

$X = \{\}$ (naive): 5.33 $\qquad \frac{|5.33 - 1.05|}{1.05} \times 100\% = 407\%$ error

$X = \{W, Z\}$ (last week): 0.85 $\qquad 19\%$ error



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Model (linear regression)

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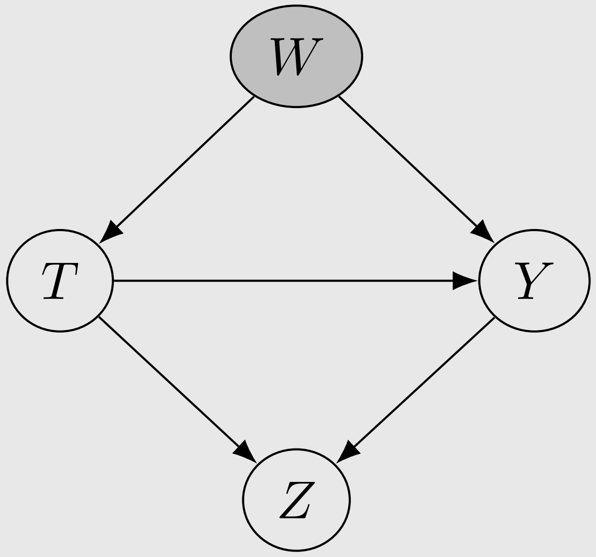
$X = \{\}$ (naive): 5.33 $\qquad \frac{|5.33 - 1.05|}{1.05} \times 100\% = 407\%$ error

$X = \{W, Z\}$ (last week): 0.85 $\qquad 19\%$ error

$X = \{W\}$ (unbiased): 1.0502



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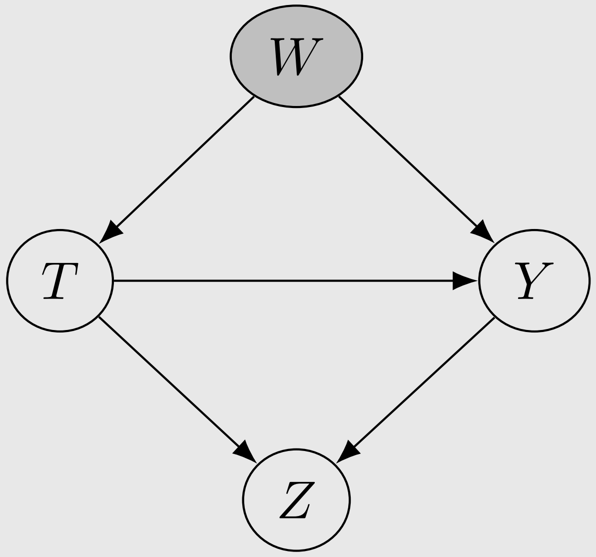
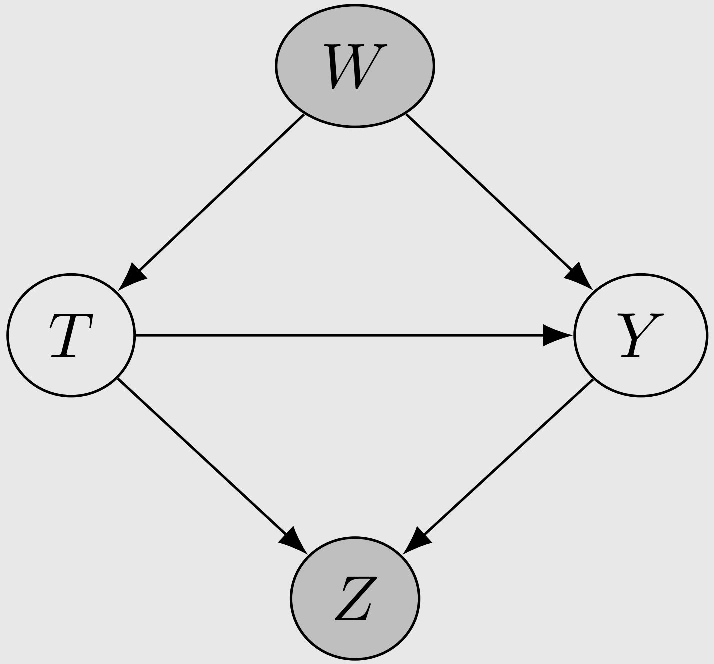
$X = \{W\}$ (unbiased): 1.0502 $\qquad 0.02\%$ error



M-bias



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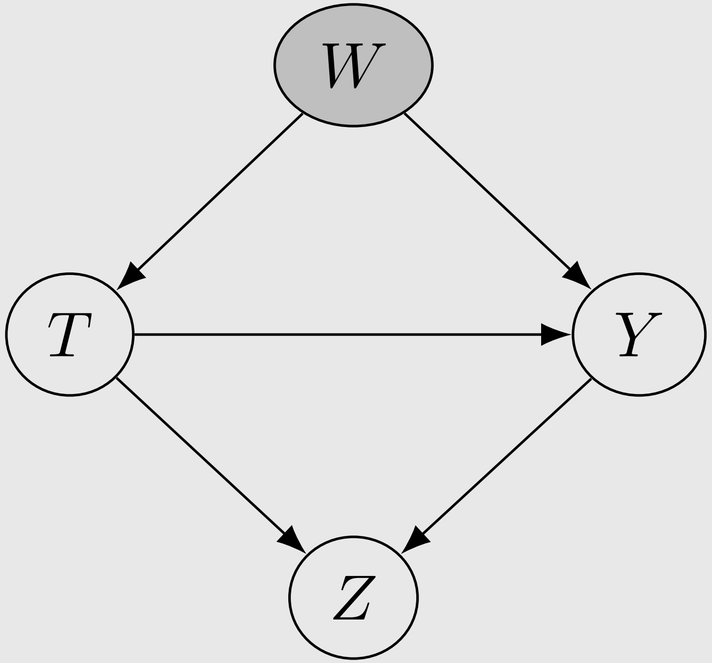

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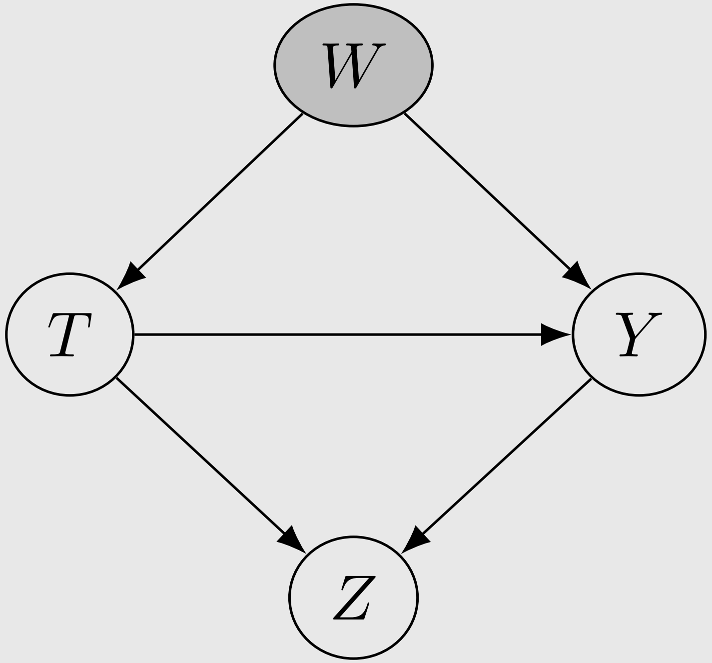

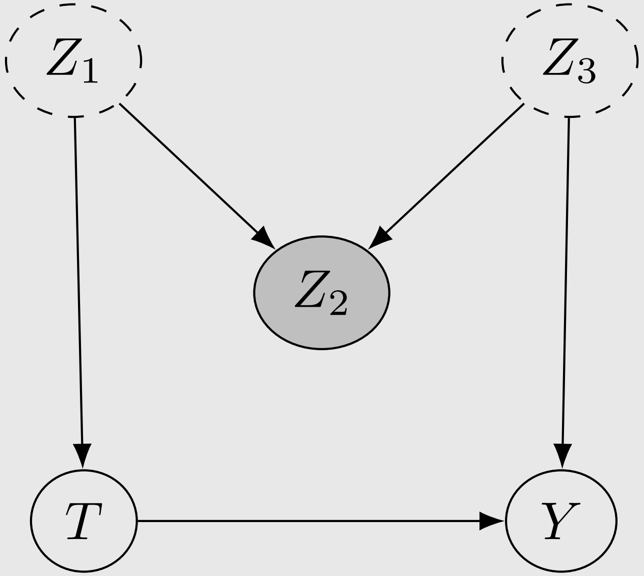

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